The Exchange Rate Effect of Multi-Currency Risk Arbitrage

Harald Hau*

University of Geneva and Swiss Finance Institute

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Abstract

Carry trade arbitrage strategies typically involve multiple currencies. Limits to arbitrage in such a setting not only slow the adjustment to the fundamental equilibrium, but can also generate transitory over- or undershooting of each exchange rate in accordance with the marginal risk contribution of each speculative position to the overall arbitrage risk. The paper uses a natural experiment to identify a particular global arbitrage opportunity and shows that arbitrage risk hedging modifies the exchange rate dynamics in the predicted manner. New spectral methods are applied to obtain a more precise inference on the cross-sectional trading pattern of the arbitrageurs.

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*GFRI, University of Geneva, 40 Bd du Pont d'Arve, 1211 Genève 4, Switzerland. Tel.: (++41) 22 379 9581. E-mail: prof@haraldhau.edu. Web: http://www.haraldhau.com. I would like to thank Bernard Dumas, Matthias Efing, Denis Gromb, Lucas Menkhoff, Michael Moore, Carol Osler, Hélène Rey, Astrid Schornik, Matti Suominen, Robert Whitelaw, and Charles Wyplosz for valuable comments on the paper. I am also grateful for various comments following presentations at the European Central bank, HEI (Geneva), Oxford University, the 5th Central Bank Conference on the Microstructure of Financial Markets in Zürich, 2009, and the EFA annual meeting, Frankfurt, 2010.

1 Introduction

Notwithstanding the importance of carry trade strategies in international finance, little is known about the structure and exchange rate effects of speculative currency trading itself.[1] This paper contributes to a better understanding in four dimensions; it (i) develops a stylized model of speculative foreign exchange (FX) trading which highlights the hedging component of any arbitrage strategy involving multiple currencies, (ii) uses the natural experiment of the global MSCI index revision (with its clearly identified currency arbitrage opportunity) to predict the optimal arbitrage strategy of hedge funds, (iii) demonstrate the quantitative importance of risk hedging for the cross-section of short-run exchange rate returns, and (iv) proposes a new spectral inference method to strengthen the statistical evidence on the predicted short-run exchange rate dynamics.

It is increasingly recognized that arbitrage occurs under frictions which may modify the validity of arbitrage relationships. The reversal to price efficiency after external shocks might be slow (Mitchell, Pedersen and Pulvino, 2007) and/or state contingent in its dependence on market funding (Brunnermeier and Pederson, 2009). Importantly, limited risk tolerance of arbitrageurs and constrained funding access may not only slow equilibrium adjustment, but give raise to new transitory asset pricing effects. In this paper we show how currencies can "overshoot" or "undershoot" because of transitory hedging demands if arbitrageurs pursue arbitrage strategies involving many currencies simultaneously. The degree of over- or undershooting is tied to the marginal risk contribution of each currency position to the overall arbitrage portfolio and can be predicted if the multi-currency arbitrage opportunity is properly identified. Hence, arbitrage frictions not only slow the equilibrium adjustment, but also imply a specific non-linear currency dynamics towards the new equilibrium.

[1] The terms 'speculation' and 'risk arbitrage' are used synonomously in line with the empirical literature on limits of arbitage and much of the investment profession. This terminology is different from the classic theoretical definition, which defines arbitrage as a riskless profit opportunity.

As a consequence exchange rate models with a direct and linear adjustment towards the fundamental exchange rate are likely to be misspecified. Such misspecification should be particularly pronounced in a world of carry trade strategies which typically involve a portfolio of currencies so that risk hedging at the portfolio level becomes an important consideration. Yet, the empirical literature on international arbitrage relationships has largely ignored such a portfolio perspective and tested arbitrage theories based on individual currency pairs. An important contribution of our paper is to show that such a restrictive approach is problematic both at the theoretical and empirical level.

Recent empirical work has linked carry trade returns to various risk factors, for example foreign interest rate spreads over the dollar money market rate (Lustig, Roussanov and Verdelhan, 2010), innovations to global FX volatility (Menkhoff *et al.*, 2011) or global liquidity factors (Brunnermeier, Nagel and Pedersen, 2009). Carry trade profits might (at least partially) be interpreted as compensation for risk. Yet the precise structure of currency speculation is not properly identified in such factor models and its effect on asset prices is difficult to disentangle from the underlying arbitrage opportunity, which is itself endogenous to monetary policy and other macroeconomic variables.[2] In consideration of these identification problems, this paper uses an event-based exogenous FX arbitrage opportunity and seeks to properly identify the structure and exchange rate effects of currency speculation relative to an (exogenous) arbitrage opportunity. The event approach gains methodological simplicity at the cost of empirical generality due to a focus on a particular data sample.

The stylized theoretical part models a multi-currency setting where risk averse currency speculators (like hedge funds) faces a price elastic currency supply in each exchange rate. How does a hedge fund optimally trade if it acquires private information about a permanent future currency demand shock? In a multi-currency setting, the hedge fund's trading

[2]For research relating uncovered interest parity violations to monetary policy see for example Grilli and Roubini (1992), McCallum (1994), Schlagenhauf and Wrase (1995), and Alvarez, Atkeson, and Kehoe (2006).

risk depends on the entire covariance structure of all currencies. A risk averse hedge fund manager should acquires positions characterized by two distinct components: The *premium component* is proportional to the expected excess return; whereas the *risk-hedging component* is (negatively) proportional to the marginal arbitrage risk of each currency position and reduces the overall risk. Importantly, such cross-sectional hedging can influence the short-run exchange rate dynamics in a complex manner. An expected premium change in one currency can alter the hedging demand in many other currencies along with their prices; thus contributing to a temporary "disconnect" between exchange rate movements and exchange rate fundamentals.[3]

The empirical part tests the theoretical framework based on an exogenous market event which allows for a clear identification of the speculators' optimal positions—including hedging demands. In December 2000, the most important provider of international equity indices, Morgan Stanley Capital Inc. (MSCI), announced publicly that it would substantially alter the composition of its global equity indices. As a consequence, many countries experienced dramatic changes in their index representation, which resulted in a reallocation of indexed equity capital from down- to upweighted currencies. A massive exogenous capital reallocation of index capital should alter the fundamental value of the respective currency pairs, unless the marginal international investor is indifferent about the currency denomination of his assets. Based on a consultation process conducted by MSCI in November 2000, informed currency speculators were able to predict cross-sectional exchange rate changes in line with anticipated capital flows and to arbitrage their exchange rate effect prior to the official announcement of the index modification. The MSCI global index revision therefore provides a unique occasion to identify an exogenous arbitrage opportunity and trace the price impact of speculative trading in the cross-sectional pattern of currency returns.

[3]See Obstfeld and Rogoff (2001), and Rogoff and Stavrakeva (2008) for a discussion of the "disconnect puzzle."

Two independent statistical strategies are used to elucidate the structure of speculative trading. First, a classical event study methodology is applied to the MSCI index revision. The cross-section of 37 spot exchange rates exhibits both the positive premium and negative risk-hedging effects. The overall explanatory power of the cross-sectional regression is substantial. Together, the premium and risk-hedging effects account for almost 55 percent of the exchange rate variation over a three-day window and more than 35 percent over a seven-day window. Excluding the risk-hedging effect from the regression reduces its explanatory power by more than half. A robustness check on a subsample of the most liquid currencies (using forward rates instead of spot rates) produces very similar results. The MSCI event returns therefore reveal that hedging arbitrage risk matters to currency speculators. Moreover, risk-hedging positions have an economically significant currency effect. The point estimates suggest that the exchange rate return difference between currencies with high and low hedging benefits (separated by two standard deviations in the hedging benefit) amounts to 3.6 percent over the five trading days of the event window. Assuming that such hedging operations by currency speculators are common practice, they could indeed contribute substantially to the short-run dynamics of exchange rates. We know of no other event study which has highlighted the empirical relevance of such FX hedging effects.

An obvious shortcoming of the conventional event study is limited statistical power if fewer than 40 cross-sectional observations are used (as is typical for exchange rate studies) and the instances of speculative trading are spread over many event days. Therefore I propose a new statistical methodology based on high-frequency data and inference in the frequency domain to obtain stronger statistical results. The intuition is as follows: Consider a group of speculators implementing the optimal multi-currency strategy; they tend to trade sequentially, but synchronized across all currencies. Any non-synchronized position built-up would sacrifice important hedging benefits associated with the portfolio approach to speculative trading. Hence, their price impact across currencies should also be extremely contemporaneous and be reflected in high-frequency comovements across different exchange rates.[4] Such high-frequency comovements can be measured as the high-frequency components of the cospectrum of exchange rate returns. For example, exchange rate pairs for which the arbitrage position is long in both exchange rates should experience positive high-frequency comovements; whereas exchange rate pairs for which the arbitrage positions are long for one and short for the other should exhibit a more negative covariance at the highest frequencies—corresponding to a negative shift of the high-frequency components of the cospectrum.

An important methodological contribution of this paper is to show that the cospectrum at the highest frequencies can be a very powerful aggregator of speculative trading patterns if speculative interventions are very synchronized across currencies or markets as can be expected under portfolio risk considerations. The high liquidity of the exchange rate market allows the use of minute-by-minute price data. The cospectrum between a pair of exchange rate returns can be aggregated into a *high-frequency band* summing up all comovements within a 15-minute interval, into a *medium-frequency band* for comovements from 15 minutes to four hours, and a *low-frequency band* capturing all remaining comovements. The spectral analysis reveals that a large share of the change in the covariance of exchange rate pairs in the 7-day arbitrage period around MSCI's pre-announcement of the index change is due to a change in the high-frequency band of the cospectrum. The event-related change in the exchange rate dynamics is characterized by strong cross-sectional return synchronicity. Moreover, the high-frequency cospectrum shift for each currency pair corresponds to the predicted arbitrage positions for the respective currency pair: The event period shift of the high-frequency cospectrum is positive if the speculative positions in both currencies have the same direction (both long or both short). The shift of the high-frequency cospectrum is

[4] A common procedure is to filter out 'high frequency' noise, as it is strongly determined by trading activity. The current study pursues the opposite objective of identifying particular patterns of cross-currency trading.

negative if optimal risk arbitrage requires speculative positions of opposite directions (one short and one long).

In the final part of this paper, I show how cospectral measures can be used to re-estimate the limited arbitrage model. Using spectral band regressions, it is possible to recover the same structural coefficients for both the premium and risk-hedging effects of arbitrage trading at much higher levels of statistical significance than in the conventional inference. The smaller standard errors allow me to make a quantitative assessment of the role of currency hedging demands on exchange rate returns. The spectral band regressions show that the (transitory) exchange rate effect of the hedging demand is at least as large as the premium effect.

In the following section, I discuss the related literature. Section 3 presents the theory and develops testable hypotheses for both spot and forward exchange rates. Section 4 discusses the MSCI index revision, its implications for the country weight changes, and the arbitrage risk related to an optimal speculative position. Cross-sectional evidence for daily spot rate returns and forward rate returns follows in section 5. Section 6 discusses the spectral methodology and corresponding evidence. Section 7 concludes.

2 Related Literature

This paper contributes to the larger literature on exchange rate behavior by focusing on the particular role of FX arbitrage trading. A better understanding of speculative hedging and its exchange rate effects may potentially reconcile two contrasting puzzles in the exchange rate literature. News, measured by a broad set of macro announcements in Andersen *et al.* (2003), generate an immediate impact on the exchange rate. But the infrequent occurrence of such public news events implies that the overall percentage of exchange rate variation explained remains very small (Evans and Lyons, 2008). Most of the daily exchange rates variation does not appear to relate to a contemporaneous major news events. Contrary to such fundamental news proxies, financial market variables capturing (directional) currency trading, such as order flow, feature a high overall correlation with contemporaneous exchange rate changes. Evans and Lyons (2002a, 2002b) document that order flow accounts for between 44 and 78 percent of the daily variation in the spot exchange rate for major currency pairs.

Speculative trading can anticipate future events and reduce the exchange rate effect around a public announcement —something which has long been recognized. Yet, speculative hedging motives and their feedback effect on the exchange rate complicate the exchange rate dynamics further. Private information about future public news in one currency can trigger the build-up and later liquidation of hedging positions (and the corresponding order flow) in correlated currencies even if those currencies are not concerned by the news event itself. A currency may over- or undershoot its equilibrium price depending on its hedging value for correlated arbitrage positions and thus appear disconnected from it own fundamentals. The event study in this paper can elucidate this important aspect of speculative trading.

Much of the literature on Uncovered Interest Parity (UIP) is concerned with providing explanations for persistent carry trade returns rather than causal inference on the effects of speculative trading on exchange rates. An exception here is Brunnermeier, Nagel, and Pedersen (2009); they provide evidence that so-called carry trades alter the distribution of exchange rate movements. The negative skewness of target currencies is interpreted as the result of a sudden unwinding of carry trades. Jylhä and Suominen (2011) explore the long-run profitability of carry trade strategies and show that the returns to carry trades have been decreasing over the last 32 years. Moreover, carry trade returns explain a significant part of hedge fund index returns.

This paper belongs to a larger finance literature on speculative trading and limited arbitrage recently reviewed by Gromb and Vayanos (2011). Market index changes have been frequently used as a suitable exogenous event to analyze speculative trading. Closely related is Hau, Massa and Peress (2011), who use the same MSCI event to document "currency price pressure effects" of capital flows. But their analysis does not encompass a portfolio approach and abstracts from all currency hedging central to the analysis in this paper. The MSCI index event is also used in Hau (2011) to study the global integration of equity markets. Similar to this paper, hedging positions are shown to matter for the arbitrageurs, but take a different form, because the equity market speculators could anticipate changes in equity betas. By contrast, the arbitrage opportunity modelled here concerns the FX market and is assumed to be proportional to the capital flow of index investors. Moreover, the statistical inference method is adapted to the small cross-section of currency observations.

An important feature of this paper is the multi-asset approach to speculation. Such a portfolio approach has previously been employed for speculative equity trading (Greenwood, 2005) and option pricing (Garleanu, Pedersen, and Poteshman, 2010). But in contrast to these papers, our framework assume that speculators face a price-elastic residual asset supply. This distinguishing model feature implies that speculators can acquire optimal hedging positions instead of just absorbing an exogenous supply shock. Thus, currency risk arbitrage amounts to net position taking, which brings the model closer to a practitioner's notion of speculation. The elastic asset supply assumption is similar to Vayanos and Vila (2007) and Greenwood and Vayanos (2010), where risk-averse speculators choose optimal arbitrage positions against a price-elastic net supply in bonds of different maturity. But unlike bond yields in their set-up, exchange rates in this paper are governed by asset-specific stochastic processes. This implies that the covariance structure of risk becomes an important element determining the optimal arbitrage position. The latter aspect is explained more formally in section 3 and distinguishes the analysis here.

3 Theory and Hypotheses

3.1 Model Assumptions

This section develops a simple limit-to-arbitrage model, in which hedge funds (or other currency arbitrageurs) take optimal speculative positions in anticipation of an exogenous currency demand shock. In the empirical section, this demand shock consists of major global index revision. Changes in index weights of stocks imply that index funds, exchange traded funds and other investors closely tracking the index mechanically adjust their international stock weights and along with it their country weights with a predictable impact on exchange rates. The timing of their rebalancing is non-discretionary and has to coincide with the index change. Hedge funds can front-run such predictable rebalancing as soon as they learn about the index revision. The model spells out the optimal trading strategy for the hedge fund in a stochastic market environment summarized as follows:

Assumption 1: Linear Stochastic Currency Supply
A currency market allows simultaneous trading in currencies $i = 1, 2, 3..., n$. *Trading occurs through a uniform price auction at (equally spaced) time points* $t = 0, \Delta t, 2\Delta t, 3\Delta t, ...T$, *with* $\Delta t = T/N$. *The (residual) liquidity supply* S_i *of currency* i *is characterized by a linear function of the exchange rate* e_{it} *(expressed in dollars per local currency) given by*

$$S_i(e_{it}) = q_i(e_{it} - \Phi_{it} + r_i t), \tag{1}$$

where $q_i > 0$ *is the liquidity supply elasticity of currency* i. *The fundamental values* Φ_{it} *of currency* i *are combined in a stochastic vector* $\Phi_t = (\Phi_{1t}, \Phi_{2t}, ..., \Phi_{nt})'$ *given by*

$$\Phi_{t=k\Delta t} = \mathbf{1} + \sum_{t=\Delta t}^{k\Delta t} \varepsilon_t, \tag{2}$$

in trading round k. *Let* $\mathbf{1}$ *denote a unit vector and innovations* $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t} ..., \varepsilon_{nt})'$ *have zero mean and a covariance* $\mathcal{E}_{t-\Delta t}(\varepsilon_t \varepsilon_t') = \Sigma \Delta t$. *The term* r_i *denotes the one-period money market interest rate in currency* i *minus the dollar money market rate.*

Assumption 1 characterizes a general exchange rate model with a stochastic vector Φ_t as the driving process.[5] The net currency supply curve is upward sloping with individual elasticities q_i. The net supply curve can be motivated empirically as a stylized representation of an aggregate FX limit order book filled with orders from both financial and non-financial agents.[6] Currency order flow moves currency prices along the liquidity supply schedule and generates persistent exchange rate effects documented in the microstructure literature. In the long run, the elasticity parameter should be related to the willingness of financial investors and non-financial firms to substitute home for foreign assets if the exchange rate becomes more favorable.[7] The linearity of the supply function is chosen for analytical convenience. For the same reason, the supply in each currency depends only on its own price and not on other exchange rates. It is also assumed that all quantities and corresponding elasticities are expressed in the same reference currency. A multi-currency demand shock denominated in the reference currency and represented by $u = (u_1, u_2, ...u_n)$ changes each exchange rate by $\frac{1}{q_i}u_i$. Since all currencies are initially normalized to 1, it is convenient to refer to the exchange rate change $e_{t+\Delta t} - e_t$ as the (approximate) exchange rate return.[8]

In the absence of any demand shock, market clearing requires $S_i(e_{it}) = 0$ for each time t and each currency i. The equilibrium exchange rate vector $e_t = (e_{1t}, e_{2t}..., e_{nt})'$ follows as $e_0 = \mathbf{1}$ for $t = 0$ and for trading rounds numbered $k = 1, 2, ...N$ as

$$e_{k\Delta t} = \mathbf{1} + \sum_{t=\Delta t}^{k\Delta t} \varepsilon_t - rk\Delta t, \tag{3}$$

[5] In a monetary model, Φ_t could capture country differences in money supply, output and interest rates.

[6] A theoretical foundation based on other financial market participants would seek to determine the elasticity parameter q_i in a rational expectation model with informed and uninformed currency traders. Alternatively, an elastic currency supply may simply be motivated by limited international asset substitutability of international portfolio investors. Even uninformative flows resulting from the global index change then generate permanent exchange rate effects.

[7] Representative agent models appear generally inconsistent with existing evidence for steep asset demand curves, as argued by Petajisto (2008). Limited market participation and short-term liquidity supply by financial intermediaries (market makers) may therefore be important market features.

[8] This amounts to a simple scale transformation of the net supply elasticity parameter.

where $r = (r_1, r_2, ..., r_n)'$ denotes the one-period foreign money market interest rate minus the dollar money market rate. By construction, the Uncovered Interest Parity (UIP) condition is fulfilled for all periods, that is[9]

$$\mathcal{E}_t(e_{t+\Delta t} - e_t + r\Delta t) = 0. \tag{4}$$

The exchange rate equilibrium is perturbed by an exogenous demand shock at time T. In the empirical part, such a currency demand shock comes from the capital reallocation of index investors (international index funds, ETFs, and international funds with a large index tracking portfolio share). Their non-discretionary mandate requires index funds to rebalance in step with the index adjustment at time T.

Assumption 2: Currency Demand Shock of Index Change
At $t = T$, a currency demand shock $u = w^n - w^o$ occurs and amounts to an exogenous capital inflow proportional to the weight changes from old index weights $w^o = (w_1^o, w_2^o, ..., w_n^o)'$ to new index weights $w^n = (w_1^n, w_2^n, ..., w_n^n)'$.

The demand shock changes the exchange rate. The exact magnitude of the exchange rate effect depends on the short-run supply elasticity captured by the parameters q_i. At time $t = T$, market clearing with $S_i(e_{iT}) = u_i$ implies

$$e_T = \Phi_T - rT + \mathbf{q}^{-1}u, \tag{5}$$

where $\mathbf{q}$ is a diagonal matrix with elements q_i and $\mathbf{q}^{-1}u = (\frac{1}{q_1}u_1, \frac{1}{q_2}u_2, ..., \frac{1}{q_n}u_n)'$. For a small q_i even modest capital flows u_i may generate a large exchange rate effect. An interesting question is to what extent the capital flows imply a permanent currency appreciation.

[9] This paper does not focus on UIP or its violation. Unlike in the Dornbusch model, the validity of UIP is not important in the model mechanism, but is assumed for expositional simplicity. It is straightforward to add a parameter $\xi_i \neq 1$ multiplying the money market rate difference r_i in eq. (1) so as to captures UIP violations.

The reduced form model here cannot address this question.[10] Instead, the focus is on the role of optimal speculative trading on the short-term exchange rate dynamics prior to the announcement of the index change.

Currency speculators are hedge funds who were informed about MSCI's pending index revision could use public information on so-called free float to predict the relative weight change of each country and therefore forecast (up to a scalar) the demand shock u. A constant absolute risk aversion (CARA) utility function for speculators generates conveniently linear demand functions in each trading interval.

> **Assumption 3: Speculators and Information Structure**
> *A unit interval of currency speculators (risk arbitrageurs) with CARA utility and a risk-aversion parameter ρ learns about the currency demand shock u at time $t = s < T$. Arbitrageurs undertake optimal arbitrage over all trading rounds in the time interval $[s, T]$. The exogenous liquidity supply functions $S_i(e_{it})$ are not affected by demand shock u or by speculative trading.*

A final consideration concerns the role of parallel markets. The currency market offers a variety of trading venues and alternatives to the currency spot market. The currency forward or futures market may represent preferred instruments of speculative arbitrage (Osler, 2008). Bjonnes and Rime (2005) document that bank dealers often prefer to route their information-based trades through the futures market. In these parallel markets, contract intermediation by a clearing house provides trade anonymity, which should be important to an informed speculator (Rosenberg and Traub, 2008). Arbitrageurs may therefore implement their trading strategy in the derivative market rather than the spot market. However, the currency forward (futures) and spot markets are highly integrated because 'covered interest parity' generally holds. For the sake of simplicity, the model abstracts from arbitrage opportunities between forward (futures) and spot markets.

[10] Evidence for persistent exchange rate effects of investor flows is provided by Froot, O'Connell, and Seasholes (2001), and Froot and Ramadorai (2005). Recent evidence on the rebalancing behavior of international equity funds suggests that portfolio managers consider home and foreign equity imperfect substitutes (Hau and Rey, 2004, 2008). This suggests that an index modification should have a permanent (or at least very persistent) exchange rate effect.

3.2 Model Solution and Hypotheses

The market clearing conditions for all trading rounds take on three different forms given by

$$\begin{aligned} S(e_t) &= 0 \quad \text{for} \quad t < s \\ S(e_t) &= x_t^A \quad \text{for} \quad s \leq t < T \\ S(e_t) &= u \quad \text{for} \quad t = T. \end{aligned} \tag{6}$$

For all trading rounds $t < s$ arbitrageurs are not yet informed about the supply shock and their speculative demand is zero. Arbitrageurs enter the market for the trading rounds $s \leq t < T$ and their optimal demand is denoted by x_t^A. In the last trading round at time $t = T$, the demand shock u occurs and trading stops.

The CARA utility assumption for arbitrageurs, together with the normality of the payoff structure, implies linear demand functions. Arbitrage between periods t and $t + \Delta t$ (with $s \leq t < T$) provides a payoff vector characterized by deviations $e_{t+\Delta t} - e_t + r\Delta t$ from the uncovered interest parity condition. The risk associated with the arbitrage is given by the covariance matrix $\Sigma\Delta t$ of exchange rate innovations. The optimal demand function of the arbitrageurs under CARA utility then follows as

$$x_t^A = (\rho\Sigma\Delta t)^{-1}\mathcal{E}_t(e_{t+\Delta t} - e_t + r\Delta t). \tag{7}$$

Repeated substitution of the arbitrage demand (7) into market clearing conditions (6) allows me to solve for the equilibrium exchange rate vector e_t backward until the period $t = s$. For trading round $t < s$ the equilibrium exchange rate follows trivially as $e_t = \Phi_t - rt$. Subtracting the fundamental value Φ_t and the interest differential rt from the exchange rate, we can define an adjusted exchange rate as $e_t + 1 - \Phi_t + rt$, which is plotted in Figure 1 as a straight line for $t < s$. At $t = s$, the exchange rate jumps by $\Delta e_s = e_s - e_{s-\Delta t}$ to the new equilibrium path determined by optimal arbitrage between $t = s$ and $t = T$. Proposition 1 characterizes the movement of the equilibrium exchange rate at $t = s$.

Proposition 1: Spot Exchange Rate Returns

Upon knowledge by the arbitrageurs at time $t = s$ of the index revision from old currency weights w^o to new weights w^n, the spot exchange rate change is positively proportional to the (elasticity-weighted) index change $\mathbf{q}^{-1}(w^n - w^o)$ and negatively proportional to the arbitrage risk term $\Sigma(w^n - w^o)$, where Σ represents the covariance matrix of currency returns. Formally,

$$\Delta e_s \approx \alpha \times \mathbf{q}^{-1}(w^n - w^o) + \beta \times \Sigma(w^n - w^o), \tag{8}$$

with $\alpha = 1 > 0$ and $\beta = -\rho(T - s) < 0$.

Proof: See Appendix for details.

The term $\mathbf{q}^{-1}(w^n - w^o)$ captures the anticipated (permanent) price impact of the index change and is referred to as the *premium component.* Arbitrage simply moves this component forward in time. The second term $-\rho(T-s)\Sigma(w^n-w^o)$ represents the additional (transitory) price impact of arbitrage risk control. It is referred to as the *risk-hedging component.* The magnitude of the latter depends on the risk-aversion parameter ρ and the duration $T - s$ over which the risk is taken. Both components affect exchange rates simultaneously when arbitrageurs learn about the index revision, as shown in Figure 1.

Proposition 1 states testable return prediction of the limited arbitrage. Arbitrage risk in the FX market is priced by a negative term $-\rho(T-s)\Sigma(w^n - w^o)$, which requires estimation of the covariance matrix Σ. We should find $\beta = 0$ only for risk neutrality of the arbitrageurs ($\rho = 0$). Second, the price impact of the weight change $w^n - w^o$ in (8) is scaled by the vector of supply elasticity $\mathbf{q}$, while the hedge term $\Sigma(w^n - w^o)$ is independent of any elasticity parameter. Intuitively, arbitrageurs choose their optimal hedge to equalize the marginal price impact of hedging across all currencies. Illiquid currencies with a very price inelastic supply will attract smaller hedge positions because hedging in these currencies is relatively more expensive. But to capture correctly the price impact of the weight change itself, a proxy for the currency specific supply elasticity q_i is needed.

The model implications for the future rate dynamics are also very simple if we assume Covered Interest Parity (CIP) and the vector of foreign interest rates (over the zero home rate) is constant at r. The price of a synthetic (arbitrage-free) forward contract for period $t + k\Delta t$ follows simply as $f_{t+k\Delta t} = e_t - rk\Delta t$. For a constant money market rate difference (money market rate in country i minus dollar rate), the forward market rate $f_{t+k\Delta t}$ should change in step with the spot exchange rate e_t, hence $\Delta f_{s+k\Delta t} = \Delta e_s$ for any k-period forward contract.

Propositions 1 characterizes the exchange rate dynamics at time $t = s$ when speculators learn about the currency demand shock. Over the consecutive interval $[s, T]$, speculators slowly liquidate their hedging positions, which should reverse the initial exchange rate effect captured by the coefficient β. But the hedge liquidation effect might extend over a longer period and is therefore more difficult to isolate empirically. The empirical strategy therefore focuses on the exchange rate effects of the speculative position build-up.

4 Data Issues

4.1 The MSCI Index Redefinition

Morgan Stanley Capital International Inc. (MSCI) is a leading provider of equity (international and U.S.), fixed income, and hedge fund indices. The MSCI equity indices are designed to be used by a wide variety of global institutional market participants. They are available in local currency and U.S. dollars (US$), and with or without dividends reinvested. MSCI's global equity indices have become the international equity benchmarks most widely used by institutional investors. By the year 2000, close to 2,000 organizations worldwide were using them. Over US$ 3 trillion of investments were benchmarked against these indices worldwide and approximately US$ 300 to 350 billion were directly indexed. The index with the largest international coverage is the MSCI ACWI (All Country World Index), which includes 50 developed and emerging equity markets. This index is also the most important in terms of its benchmark status.[11]

On December 10, 2000, MSCI formally announced that it would adopt a new policy of stock weight calculation based on so-called 'free-float' weights. The equity index would adjust by 50% towards the new index on November 30, 2001 and the remaining adjustment was scheduled for May 31, 2002. Free-float weights take into account pyramid ownership and control structures in many different countries. Free-float weights better reflect the limited investability of many stocks and therefore entire countries. However, the formal announcement of the adoption was preceded by an internal decision process and accompanied by a consultation process with the investment community. The first relevant date in this respect dates back to February 2000, when MSCI communicated that it was reviewing its policy on index weights. On September 18, the competing index provider Dow Jones adopted free-float weights, increasing the pressure on MSCI to take a decision. The next day, MSCI published a consultative paper on possible changes and elicited comments from the investment community. Any adoption decision would be based on the feedback from its clients.

The next important event occurred on December 1, 2000, when MSCI announced that it would communicate its decision nine days later, on December 10. This pre-announcement presented a strong signal to arbitrageurs that MSCI had taken a decision about the weight change and that public announcement of the index revision was imminent.[12] The strongest

[11] Less important subindices are the MSCI World Index (based on 23 developed countries), the MSCI EM (Emerging Markets) Index (based on 27 emerging equity markets), the MSCI EAFE (Europe, Australasia, Far East) Index (based on 21 developed countries outside of North America), and the MSCI Europe. All subindices are composed of subsets of stocks in the MSCI ACWI and are therefore similarly affected by an overall change to the index methodology. As long as the index equity invested in subindices is small in comparison to the equity indexed to the benchmark MSCI ACWI, the free-float redefinition of the subindices should not dramatically modify the overall index flows.

[12] The actual announcement on December 10 seemed to have confirmed market expectations. Commentators remarked that MSCI's adoption decision was broadly in line with the previous consultation paper. Only the target level of 85 percent of the national market was somewhat higher than expected (by five percent) and the implementation timetable was longer than most observers had expected. See the investment newsletter, 'Spotlight on: Throwing Weights Around', Hewitt Investment Group, December 2000.

arbitrage activity can therefore be expected to occur around this date. Supportive evidence for this interpretation comes from data on the Euro/Dollar spot trading volume in the electronically brokered EBS and Reuters D2000 trading platforms. The first trading day after the pre-announcement (Monday, December 4) is characterized by very large spot trading volume, exceeding the daily average volume by 30 percent.[13] Some arbitrageurs are likely to have anticipated the free-float adoption earlier than December 1, and acquired their arbitrage positions before this date. Information leaks from MSCI or rational anticipation after the consultation process may have informed arbitrageurs about the likely free-float adoption. Interestingly, spot trading volume in the Euro/Dollar rate peaked on November 30, 2000, and was 32.5 percent above its quarterly average.

By the end of December 4, 2000—the first trading day after the pre-announcement—the speculative position build-up of hedge funds should have been largely accomplished. The end of this trading day therefore marks the end of the event window. By contrast, the exact beginning of the arbitrage activity is more difficult to date. In order to deal with this issue, alternative starting dates for the event windows are used—covering three, five or seven trading days up to December 4, 2000. The two larger windows start on November 24 and 28, respectively. These longer windows may capture trading by a larger group of privately informed hedge funds. Yet an excessively early start date for the event window (without early arbitrage trading) should bias the results against finding strong cross-sectional return patterns.

[13] I examine transaction volumes in the Euro/Dollar spot market available for the period 01/08/2000 − 24/01/2001. Euro/Dollar spot market volume surge should accompany any major international equity reallocation. The data combine all electronically brokered spot contracts in both the EBS and Reuters D-2000 trading platforms on any given day. The first trading day after the pre-announcement (Monday, December 4) is characterized by very large spot trading volume of 17,610 contracts. It exceeds the daily average volume by 4,051 contracts, or 30 percent. By contrast, trading volume on Monday, December 12, − the first trading day after the second announcement − was below average. The transaction volumes indicate that December 1, 2000 was the relevant news. We thank Paolo Vitale and Francis Breedon for generously providing the transaction data.

4.2 Index Weight Changes and the Arbitrage Risk

The new stock selection criterion based on free-float had drastic consequences for the weight of different currencies in the global MSCI index. The absolute weight change $w^n - w^o$ has a standard deviation of 0.006 and appears small. An alternative measure is the percentage weight change defined as the weight change $w^n - w^o$ divided by the mean $\frac{1}{2}(w^n + w^o)$ of old and new weights. The percentage weight change features a standard deviation of 0.326 and is substantial. Figure 2 plots the percentage weight change for each country as a function of the old weights expressed in logs. The Euro area countries are aggregated to a collective currency weight (Euro area) since these share a common currency.

Figure 2 illustrates that a large number of countries and currencies experienced a dramatic reduction in their index representation. For no fewer than 10 countries the aggregate percentage weight loss exceeded -70 percent (Argentina, Chile, Colombia, Czech Republic, India, Malaysia, Pakistan, Thailand, Turkey, Venezuela) because of a large market share of stock companies with investment restrictions. Most other currencies also experienced index weight losses. The largest absolute weight decreases were registered for Brazil, India, Taiwan, Hong Kong and Mexico. Only eight countries showed a positive country weight change, namely Australia, Egypt, Finland, Greece, Ireland, Morocco, the United Kingdom, and the United States. Particularly large and positive were the weight changes for Ireland, the United Kingdom and the United States with percentage weight increases of 11.4 percent, 10.9 percent and 12.0 percent, respectively.

The model of limited arbitrage developed in section 3 implies that speculators adjust their arbitrage portfolio weights not only to the expected premium proxied by $q^{-1}(w^n - w^o)$, but also scale their portfolio weights inversely to the marginal risk $\Sigma(w^n - w^o)$ of each currency position over the arbitrage period. This requires an estimation of the (expected) covariance matrix Σ. Assuming intertemporal stability in the correlation structure, the covariance matrix is simply estimated using two years of daily FX data from July 1, 1998 to July 1, 2000. The exchange rate data are based on end of the day mid-price quotes in London available from Datastream. The estimation period ends four months prior to the announcement date of December 1, 2001. Thus the covariance estimate is not affected by the actual event. The total sample includes 37 currencies.[14] Table 1, Panel B, provides a summary of the marginal arbitrage risk contribution $[\Sigma(w^n - w^o)]_i$ of the 37 sample currencies. Marginal risk contributions are negative for most currencies. Increasing the portfolio weights in these currencies provides a hedge against the exposure from excessive dollar investment implied by the strong weight increase for the U.S. currency.

On a conceptual level, historical data certainly provide an imperfect measure of the forward looking covariance matrix. But to avoid the estimation issues with a high-dimensional GARCH model, we just assume that the historical sample covariance estimates the forward looking covariances reasonably well. A second (more relevant) robustness issue concerns the choice of data frequency. Does a weekly sampling frequency alter estimates of the marginal risk contribution of each currency? To explore this issue the estimation procedure is repeated with weekly spot rate return data. The correlation between estimates based on daily and weekly sampling is 0.984. Optimal arbitrage portfolios therefore look reasonably similar independently of the sampling frequency of the historical data. Robustness was also checked for a change of the sample period to three years and to 18 months. Underlying this relative robustness is the fact that the marginal risk terms only involve the estimation of a $n = 37$ dimensional vector $\Sigma(w^n - w^o)$ even though Σ has much more free parameters.[15]

[14] The 13 countries in the Euro area share a common exchange rate after January 1, 1999. Prior to this date, the ECU currency basket is used. Argentina and Malaysia feature only very incomplete exchange rate data over the estimation period; both countries are excluded from the analysis. China and Hong Kong also stand out with currencies of very low exchange rate variation because of their peg to the U.S. dollar. Neither country was excluded from the analysis. However, excluding both countries makes no qualitative difference to the overall results. Our results are also robust to excluding Turkey—a country experiencing a currency crisis in 2001.

[15] While we estimate the $(n^2 - n)/2 + n$ parameters of Σ for $n = 37$, multiplication by $w^n - w^o$ averages the estimation error of any single element. Measurement error with respect to the regressor $\Sigma(w^n - w^o)$ should therefore not be a serious problem for the analysis. Under large measurement error, we would have to adjust

4.3 Cumulative Event Returns

Before undertaking a more formal regression analysis, it is interesting to examine the time series behavior of exchange rates around the announcement of the MSCI index change. For this purpose, I double sort the 37 currencies by $q_i^{-1}(w^n - w^o)_i$ and $\Sigma(w^n - w^o)_i$ with $q_i = \frac{1}{2}(w^n - w^o)_i$.The 18 currencies with a percentage weight increase larger than the median are sorted into a group labeled $W+$, while 19 of the most downweighted currencies are labeled $W-$. On a second sort, the currencies in each group are ranked by their marginal arbitrage risk $\Sigma(w^n - w^o)_i$ into the nine currencies with the lowest arbitrage risk and therefore highest hedge benefits $H+$ and the nine (or 10) remaining currencies with low hedge value labeled $H-$. Four groups of currencies $W+H+$, $W+H-$, $W-H+$, and $W-H-$ are thus obtained. Their respective cumulative average (equal-weighted) return for each group is plotted in Figure 3.[16]

According to the risk arbitrage theory developed in section 3, currencies in group $W+H+$ are the most attractive for speculative long positions and those in group $W-H-$ are the most attractive for short positions. Figure 3 shows the predicted cumulative return pattern whereby currencies in the group for long positions tend to appreciate prior to the announcement event on December 1 as opposed to those most suitable for short positions. The average return for the most desirable currencies in group $W+H+$ increases by more than three percent over the seven trading days from November 24, 2000 to December 4, 2000. Currency returns in group $W-H-$ over the same interval feature a negative average return of 120 basis points. Much of the difference in the cumulative return starts to emerge before the first announcement on December 1, 2000 and suggests arbitrage trading prior to this

the confidence intervals of the second stage regression.

[16] Grouping currencies according to median percentage weight change provides two equally large subsamples. The focus is on their relative performance. Censoring at the absolute zero weight change is not useful. The arbitrage theory does not state in which currency (US$ or some other currency or currency basket) arbitrageurs define their objective function. This means that any exchange rate effect can only be predicted up to a fixed effect common to all currencies. Only relative currency effects are of interest here.

date. The strong increase of the cumulative average return for currencies in group $W+H+$ relative to group $W-H-$ from November 20, 2000 to December 4, 2000 validates the event window selection. This period includes seven trading days and represents the largest of the three event windows considered.

5 Cross-Sectional Evidence

The portfolio approach to risk arbitrage developed in section 2 provides cross-sectional exchange rate predictions for the arbitrage event. First, an exchange rate appreciation ($\Delta e_{it} > 0$) of currency i is positively related to its weight change $(w^n - w^o)_i$ in the portfolio of the global investor. Second, the exchange rate appreciation of a particular currency is negatively affected by its risk contribution to the arbitrage strategy. The following section tests these sign restrictions using a linear panel regression.

5.1 Evidence on the Spot Rate

The natural correlation of exchange rates suggests a correlated panel approach with one equation for each currency. The linear model is given by

$$\Delta e_{it} = \lambda_0 + \lambda_1 \times D_t + \alpha \times D_t \times \frac{1}{q_i}(w^n - w^o)_i + \beta \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it}, \qquad E(\mu_t \mu_t') = \Sigma, \quad (9)$$

where the daily (log) exchange rate change Δe_{it} in currency i is regressed on a constant; an event window dummy D_t marking alternatively a three, five or seven trading day event window around the announcement day of December 1, 2000; the elasticity-weighted currency weight change $\frac{1}{q_i}(w^n - w^o)_i$ interacted with the event dummy; and the marginal arbitrage risk contribution $[\Sigma(w^n - w^o)]_i$ of currency i interacted with the event dummy. Two different parameter sets are used for the currency supply elasticities q_i. The first specification proxies the currency supply elasticity with the MSCI stock market capitalizations. The midpoint $q_i = \frac{1}{2}(w^n + w^o)_i$ between the new and old market weights is taken as the measure of (relative) market capitalization. The term $\frac{1}{q_i}(w^n - w^o)_i$ then represents the percentage change of a country's index representation plotted in Figure 2. A second elasticity specification is based on FX trading volume in each currency, that is $q_i = Vol_i^{FX}$. The currency-specific trading volume is obtained from the BIS trianual market survey of 2001.[17] Countries with large equity markets tend to have highly liquid currency markets. This is illustrated by the high correlation of 0.943 between the capitalization-based proxy of currency market liquidity and the volume-based proxy. Both scaling variables q_i should therefore produce similar results. By contrast, the correlation between the arbitrage risk measure and the scaled weight change is small at 0.29 and 0.21 for the capitalization and volume elasticity proxy, respectively. Regressor colinearity is therefore not a concern.

A general cross-currency correlation structure is allowed for the error term μ_{it}. To estimate this error structure more precisely, I use not only the daily data of the event window, but supplement the event window by two years of exchange rate data from July 1, 1998 to July 1, 2000. For this period prior to the arbitrage activity, the dummy variable D_t takes the value of zero. Only for the event window is the dummy variable 'switched on,' capturing the return chasing component through the coefficient α and the hedging component through the coefficient β. The model in section 3 predicts $\alpha > 0$ and $\beta < 0$. For the special case where the FX arbitrageurs are risk neutral ($\rho = 0$), the exchange rate effect captured by the coefficient β should be insignificant. All currencies are expressed in dollar terms where $\Delta e_{it} > 0$ denotes the dollar depreciation or foreign currency appreciation. The constant term λ_0 captures the average long-run dollar depreciation against all other currencies, while the coefficient λ_1 estimates the average dollar depreciation over the event window only. Any

[17] Unfortunately, the currency specific trading volume is not available for all currencies in the BIS survey. Where such data are missing I extrapolate the currency trading volume from the FX trading volume undertaken in the respective country, which is highly correlated with the trading volume of its currency.

particular dollar movement against all other currencies may simply represent a U.S.-specific effect and is therefore difficult to interpret. Translating all exchange rate returns into an alternative currency than the dollar (or into a currency basket) amounts to adding a fixed effect and should only modify the coefficients λ_0 and λ_1.

Table 2 presents the regression results for the 37 spot rates in the sample. Regressions in Panel A proxy exchange rate elasticities by the average MSCI index representation of a currency, while Panel B reports analogous results for currency elasticities proxied by FX trading volume. To evaluate the robustness of the findings, regression results are reported for event windows stretching alternatively over three, five and seven days. First, a baseline regression that excludes the arbitrage risk term is reported. A second specification includes the price effect of the hedging demand. The coefficient λ_0 is of no particular interest and not reported.

In the reduced specification in Panel A, the premium effect $\frac{1}{q_i}(w^n - w^o)_i$ enters statistically significantly at a one percent level for all three event windows. But this specification does not control for the risk-hedging demand of the arbitrageurs and may therefore be miss-specified. Inclusion of the risk-hedging term reduces the significance level of the premium term. It remains positive, but statistically significant only for the 7 day window. The risk-hedging demand, on the other hand, has the predicted negative sign at high levels of statistical significance. The adjusted R-squared substantially increases for all windows under inclusion of the risk-hedging term. For example, the five-day window in Panel A features an adjusted R-squared of 0.43 $-$ an impressive empirical fit for an exchange rate model. Currencies with very low and negative marginal risk contributions experience a relative appreciation. A decrease of the arbitrage risk of a currency by one standard deviation (= 0.005) implies an average daily currency appreciation of 0.36 (= 0.005 $\times$ 71.85) percent, or 1.80 percent over the five trading days. A comparison of the (relatively imprecise) point estimates $\widehat{\alpha}$ and $\widehat{\beta}$ suggests that the hedging effect on currencies is large compared to the premium effect.

The point estimates of $\widehat{\beta}$ in Panel A are between 65 and 340 times larger than $\widehat{\alpha}$, while the standard deviation of its regressor is only 65 times smaller. But large standard errors on the coefficients prevent any strong quantitative conclusion.

Panel B reports results for the specification where exchange rate elasticities are proxied by trading volumes. Overall, this alternative specification provides very similar results. The coefficient on the index rebalancing price effect $\frac{1}{q_i}(w^n - w^o)_i$ is positive, but statistically insignificant, while the exchange rate impact of the hedging demand $[\Sigma(w^n - w^o)]_i$ again has the correct negative sign and is statistically significant at the conventional one percent level. As before, the empirical fit of the model is vastly superior for the full specification. The low level of statistical significance for the premium term may be either due to measurement error with respect to the elasticity parameter or to the small sample size of only 37 observations. Also, some of the currencies in this sample are relatively illiquid and may therefore have been excluded from arbitrage trading. The next section focuses on the 22 more liquid currencies.

5.2 Evidence on the Forward Rate

Speculative positions can be acquired either in the underlying spot market or in the forward (futures) markets. Speculators may prefer derivative markets to engage in FX arbitrage (Osler, 2008). In this section I verify whether the results obtained for currency spot rates extend to the forward market. Forward rates are available from Reuters (via Datastream) as the 4.00 pm U.K. interbank closing rate for the most common maturities of one week, one month, three months and six months. The daily forward rate data for these maturities are available for 22 out of 37 currencies. The 22 quoted rates represent the most liquid forward rates.

Before estimating the model implications for forward rates, it is useful to examine the relationship between the different forward rates and the spot rate. Forward rates are generally highly correlated with spot rates and the event period in this study is no exception.

The correlation of the spot rate return and the forward rate return at the daily frequency over the seven day event window is above 0.99 for all four forward rate maturities (one week, one month, three months and six months). This extremely high correlation leaves little or no scope for any differential reaction of forward market rates and spot rates to speculative buying pressures. Under validity of the covered interest rate parity condition, this also implies that interest rate differences between home and foreign money market rates were not significantly affected by currency speculation.

The extremely high correlation between the spot rate and forward rate returns make these variables almost interchangeable as event return measures. However, the forward rate sample covers a subset of the most liquid currencies. They represent the 22 with the highest trading volume. The extremely high liquidity in these currencies attenuates the price pressure for any given position size, but should simultaneously increase a currency's suitability for large hedging positions.[18] Repeating the cross-sectional regression for the sample of 22 forward rates presents a useful robustness check for highly liquid currencies.

The linear panel specification remains identical, given by

$$\Delta f_{it} = \lambda_0 + \lambda_1 \times D_t + \alpha \times D_t \times \frac{1}{q_i}(w^n - w^o)_i + \beta \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it} \qquad E(\mu_t \mu_t') = \Sigma, \tag{10}$$

where $\Delta f_{it} > 0$ represents a daily forward rate rise. The event period window, with its short time series of either three, five, or seven trading days, is again complemented by two full years of forward rate return data to obtain an improved estimate for the cross-sectional correlation structure of exchange rates. The event dummy D_t takes on the value of 1 for the event window and is zero otherwise.

Table 3 reports the regression results for the one week forward rates for the two elasticity specifications. Panel A proxies elasticities by the MSCI market weights and Panel B by FX

[18] In our linear model, both effects cancel so that the price effect of the hedging component $[\Sigma(w^n - w^o)]_i$ is independent of the price elasticity parameter q_i.

trading volumes. As in Table 2, the premium term $\frac{1}{q_i}(w^n - w^o)_i$ has a positive sign and the risk-hedging term $[\Sigma(w^n - w^o)]_i$ the expected negative sign. The premium term is now statistically significant at the five percent level for all three event windows. The adjusted R-squared for the smaller sample of 22 forward rates is still higher than for the larger sample of 37 spot rates. For the three-day window the adjusted R-squared exceeds 57 percent. This shows that the model has a much better fit when we focus on the most liquid currencies. The same regression is repeated with one week and three-month forward rates and the results are very similar (but not reported). In conclusion, the currency hedging components of speculative trading can be found both in spot rate and forward rate returns. Speculative risk hedging demands are price relevant for the entire sample of all currencies, but also for the subsample of the most liquid currencies.

6 Spectral Implications of Multi-Currency Arbitrage

While the statistical evidence in the previous section supports the model and provides a very good empirical fit to the data, it falls short of providing high statistical significance levels for the model parameter values. This is not surprising, given the small number of exchange rates on which the model is tested. Exchange rates are notoriously volatile and even a relatively short event window incorporates many other confounding exchange rate effects that cannot be controlled for and hence enter the error terms of the regression. The following section develops a new methodology that extracts the speculators' trading pattern at the microstructure level by focusing on the high-frequency return cospectrum of currency pairs.

The model described in section 3.1 assumed for simplicity that the speculative position build-up occurred synchronized across all currencies and simultaneously for all speculators. More realistically, different speculators acquire their speculative positions at different moments and build the overall speculative position gradually to minimize price impact. Nevertheless, speculators want to avoid an unbalanced position, which would scarify hedging benefits. Hence, even under a stepwise position acquisition, cross-sectional trading should still occur in highly synchronized manner. This latter feature implies a particular cross-sectional return pattern, even if speculative action is uncoordinated across arbitrageurs. The spectral analysis provides a magnifying glass to focus on these joint high-frequency dynamics of exchange rate pairs and thus identifies speculative trading more distinctly.

6.1 Methodology and Data

Next, I develop the spectral implications of multi-asset risk arbitrage strategies in more detail. If risk management is an important element of multi-asset risk arbitrage, then *synchronous* implementation of the speculative positions across all currencies should be common practice. Comovement in statistics is generally captured by covariance and can be decomposed into its different frequency components, or cospectrum. Formally, the covariance between two demeaned time series X_s and Y_s has the frequency domain representation

$$Cov(X_s, Y_s) = \frac{1}{S}\sum_{s=1}^{S} X_s Y_s = \frac{1}{2}\sum_{f=1}^{N}\left(\alpha_f^X \alpha_f^Y + \beta_f^X \beta_f^Y\right) = \sum_{f=1}^{N} Cosp(X, Y, f), \tag{11}$$

where (for an odd number S) each of the $N = (S-1)/2$ cospectrum terms $Cosp_{XY}(f)$ represents the contribution of comovements at frequency f to the total comovement or covariance.[19] The cospectrum terms follow directly from a discrete Fourier transformation of the individual series X_s and Y_s, where α_f^X and α_f^Y are coefficients of the cosine components and β_f^X and β_f^Y are the coefficients of the sine components. The additivity of the cospectrum allows the definition of spectral bands that aggregate certain frequencies in a frequency band. For the purpose of the analysis, I define three different spectral bands $B = \{$High, Medium,

[19] For details, see Hamilton (1994), page 275.

Low}, which decompose the covariance of any pair (i, j) of exchange rate returns Δe_{is} and Δe_{js} into its high-, medium-, and low-frequency cospectrum; hence

$$Cov(\Delta e_{is}, \Delta e_{js}) = \sum_{f=1}^{N} Cosp(i, j, f) = \sum_{B=H,M,L} Cosp(i, j, B). \tag{12}$$

This decomposition can be applied to any sample of currency pairs. The double sorting of currencies from section 4.3 into (relatively) up- and downweighted $(W+/W-)$ and those with positive or negative hedge value $(W+/W-)$ is useful again. Currency pairs (i, j) drawn from the most desirable arbitrage currencies $(W+H+)$ should be subject to synchronized joint buying by speculators, which should generate an increase in the high-frequency co-movement measured by the high-frequency cospectrum band $Cosp(i, j,\text{High})$. Synchronized selling of two currencies from the group of least desirable arbitrage currencies $(W-H-)$ should also generate a positive high-frequency return co-movement, as both currencies are expected to show a negative return due to joint short selling. However, (cross group) currency pairs, where one currency is drawn from the group $(W+H+)$ and the other from the group $(W-H-)$, should show a more negative co-movement for the event period. To compare the cospectrum of the event period to the ordinary cospectrum at regular times, I use a control period of the same length as the event period and define the change in the cospectrum, or cospectral shift, of the frequency band B as

$$\Delta Cosp(i, j, B) = Cosp(i, j, B)^{Event} - Cosp(i, j, B)^{Control}, \tag{13}$$

where $Cosp(i, j, B)^{Event}$ and $Cosp(i, j, B)^{Control}$ denote the cospectrum of the event and control period, respectively. The remainder of the analysis focuses on changes in the cospectrum relative to the natural cospectrum for any currency pair. Currency pairs with arbitrage positions in the same direction (both long or both short) should be characterized by a positive spectral shift in the high-frequency cospectrum, while currency pairs with arbitrage positions in opposite directions (one long, the other short) should feature a negative shift in the high-frequency cospectrum.

> **Proposition 2: Synchronous Implementation of Arbitrage Positions**
> *Synchronous trading across currencies by risk arbitrageurs implies a specific modification of the high-frequency components of the cospectrum for each currency pair (i, j). For the event period, the shift in the cospectrum $\Delta Cosp(i, j, B)$ is (i) detectable in the highest spectral band $B = H$ and (ii) proportional to the product $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$ of predicted event period exchange rate returns, where $\Delta\widehat{e}_i = \alpha \times q_i^{-1}(w^n - w^o)_i + \beta \times [\Sigma(w^n - w^o)]_i$.*
> Proof: See Appendix for details.

Applying Proposition 2 requires high-frequency data for a cross-section of exchange rates. These data are obtained from the commercial data provider Olsen Associates for the three months from September to December 2000. Olsen systematically records the best FX bid and ask quotes from the Reuters terminal at one-minute intervals. Such quotes are 'firm' and can be executed by other market participants. High-frequency data were available for all the 37 currencies except China and Sri Lanka. The spectral analysis is based on midprices calculated as the arithmetic average of the best bid and ask price at the end of each minute interval. If no bid or ask price is available for a one-minute interval, the last available quote is used to calculate the midprice. The event period is given by the same seven trading days from November 24 to December 4 for which the cross-sectional evidence was presented. As control period I use the period from September 8 $-$ 18, 2000, which also covers seven trading days. Saturdays and Sundays are excluded because of low trading intensity during the weekend.[20]

Both the event or arbitrage period and the control period comprise a total of $10,080$ $(= 60 \times 24 \times 7)$ one-minute intervals. A large number of intervals do not feature new price quotes. In this case it is assumed that the previously quoted bid and ask are still valid and the

[20] The trading days are November 24, 27, 28, 29, 30, and December 1 and 4 for the event period and September 8, 11, 12, 13, 14, 15, 18 for the control period, respectively. Both series start on a Friday morning and end on a Monday night. I also experimented with an alternative seven day control period and obtained similar results.

midprice is therefore unchanged. Highly liquid markets like the Euro-Dollar rate or the Yen-Dollar rate have new quote arrivals for approximately 90 percent of the one-minute intervals. At the other end of the scale we find the Egyptian pound with new quotes in its dollar rate in only one percent of the $10,080$ intervals. A second feature of the high-frequency exchange rate returns is their high negative autocorrelation for lags up to five minutes. Pooling all exchange rates produces a partial autocorrelation at lag 1 of -0.258 (-0.237) for the event (control) window and -0.105 (-0.088) at lag 2. The strong negative serial correlation of the midprice indicates short-run reversal of the price impact of trades. Intuitively, demand shocks remove some of the liquidity supply on one side of the market and it may take time for a new best quote to replace the absorbed liquidity. Importantly, such strong negative serial correlation due to trading events will tend to leave a particularly large footprint in the high-frequency spectrum of the exchange rate return process.[21] It also implies that the cospectrum of currency pairs is particularly pronounced at the highest frequencies if both currencies experience simultaneous trading action. The speculative multi-asset trading strategies should therefore be most detectable at high frequencies and this motivates the spectral analysis.

The high-frequency band is defined as the sum of the 15 highest frequencies, which capture return synchronicity within a 15-minute interval. The 15-minute interval is motivated by possible execution delays for portfolio strategies which, according to currency traders, can amount to a few minutes in some less liquid currencies. The next 225 highest frequencies are aggregated into the medium-frequency cospectrum, thus representing comovements between 15 minutes and four hours. All $5,160$ lower frequencies are aggregated into the low-frequency cospectrum. In order to make the cospectrum more comparable across frequency bands of different sizes, it is useful to scale the cospectrum band by the number of frequencies it com-

[21] *Positive* serial correlation of an MA(1) or AR(1) process, for example, implies a spectral density function that is decreasing in the frequency spectrum, while *negative* serial correlation implies a spectral density function that is increasing for higher frequencies.

prises. The 'scaled cospectrum' then captures the average covariance contribution of a single 'representative' frequency within the band. The exact segmentation of the frequency band is somewhat arbitrary. However, the results reported in the following section are robust to alternative (though qualitatively similar) segmentations of the frequency spectrum.

6.2 Spectral Evidence for Currency Groups

Examining the cospectral shift for selected currency pairs allows a first look at the properties of return co-movement across the various frequency bands. The most attractive buy (or long) currencies for a speculator are combined in the portfolio $W + H+$ and the least attractive sell (or short) currencies in the portfolio $W - H-$; the two other groups $(W+H-, W-H+)$ are ignore in this section. For currency pairs (i, j) where both currencies are drawn from the same portfolio, we should observe a positive change in the high-frequency cospectrum due to simultaneous buying or selling. By contrast, a negative co-movement in the high-frequency cospectrum is expected if currency i is drawn from portfolio $W + H+$ and currency j from portfolio $W - H-$.

Table 4 reports the evidence on the cospectrum for currency pairs formed within groups $W + H+$ and $W - H-$ in Panel A and across groups in Panel B. The cospectrum here is scaled for the number of frequencies entering each spectral band. It is stated separately for the event period, the control period, and the spectral difference between both. The Wilcoxon sign-ranked test reports if the difference in the cospectrum is statistically significant for any of the three spectral frequency bands. In Panel A, a positive cospectrum is found for both event and control periods at all three frequency bands. Interestingly, the spectral change is concentrated in the high-frequency band and features the expected positive sign. The Wilcoxon sign-rank test here strongly rejects the hypothesis that the median is the same for the event and control sample. Currency pairs, for which joint buying or joint selling is the optimal arbitrage strategy, clearly show a stronger high-frequency co-movement over the event period. For the remaining two spectral bands, the change in the cospectrum is statistically insignificant. In Panel B, the high-frequency cospectrum also features the largest change between event and control period. The high-frequency cospectral shift is negative as expected, since the currency pairs in Panel B combine exchange rates for which the optimal arbitrage portfolio prescribes long and short positions. All other spectral bands have a cospectrum change that is not significant at the one percent level.

Panel C reports additional sign tests for the relationship between pair type (within or across groups) and the direction of the cospectral shift. For the high-frequency band, the 68 within-group currency pairs show an increased cospectrum in 45 cases, while the 72 cross-group pairs show an increased cospectrum for only 27 cases. The Fisher test indicates a clear statistical association between pair type and the direction of the cospectral shift. No such sign correlation is detectable for the lower frequency bands at the conventional one percent confidence level. The particular return pattern of arbitrage trading becomes most visible in the high-frequency domain. A graphical illustration of the evidence in Table 4 is presented in Figure 4. The change in the covariance within and across groups is concentrated in the high-frequency band that measures comovements within a 15-minute interval. The non-parametric test illustrate a large improvement in statistical power for detecting cross-sectional trading patterns due to an analysis in the (high) frequency domain.

6.3 Spectral Band Regressions

In the final section of the paper, I show how cospectral measures can be used to infer structural parameters. According to proposition 2, the high-frequency cospectral shift for each currency pair (i, j) is proportional to the product $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$. This allows a spectral band regression based on all currency pairs, which greatly increases the sample size. Spectral band regressions have been advocated by Engle (1974) for the analysis of macroeconomic data. But they represent an even more powerful tool for the analysis of financial data, which are often available as high-frequency panel data. Spectral band regressions can detect cross-sectional return patterns induced by multi-asset portfolio choice.

Building on the limited arbitrage model in section 3, the expected exchange rate change in currencies i and j is linear in the two parameters α and β according to

$$\Delta\widehat{e}_i(\alpha,\beta) = \alpha \times q_i^{-1}(w^n - w^o)_i + \beta \times [\Sigma(w^n - w^o)]_i \tag{14}$$

$$\Delta\widehat{e}_j(\alpha,\beta) = \alpha \times q_j^{-1}(w^n - w^o)_j + \beta \times [\Sigma(w^n - w^o)]_j, \tag{15}$$

where $q_i^{-1}(w^n - w^o)_i$ again represents the premium effect and $[\Sigma(w^n - w^o)]_i$ the risk-hedging effect. The change in the cospectrum $\Delta Cosp(i,j,B)$ is explained by the quadratic form $\Delta\widehat{e}_i(\alpha,\beta) \times \Delta\widehat{e}_j(\alpha,\beta)$, hence ($\gamma > 0$)

$$\Delta Cosp(i,j,B) = \gamma \times \Delta\widehat{e}_i(\alpha,\beta) \times \Delta\widehat{e}_j(\alpha,\beta) + \epsilon_{ijB} \qquad \text{for} \qquad B = \text{High, Medium, Low.} \tag{16}$$

It is straightforward to estimate this quadratic model using a maximum likelihood method. As for the event return, the premium effect has a positive coefficient, hence $\alpha > 0$, and the risk-hedging effect a negative coefficient, thus $\beta < 0$.[22] But the left-hand side variable is now given by the sample cospectrum, which increases the number of observations to all currency pairs and furthermore allows a separate regression for each spectral band. Under simultaneous implementation of the arbitrage strategy, the best regression fit is expected for the highest frequency band. The high-frequency band also aggregates the smallest number of Fourier coefficients. Only the sine and cosine coefficients of the 15 highest frequencies are used, which implies a total of 1050 ($= 2 \times 15 \times 35$) Fourier coefficients for 35 currencies. The 1050 Fourier coefficients fully characterize the high-frequency behavior of all 35 exchange rate return series. Aggregation of these Fourier coefficients into 595 ($= 35 \times 34/2$) different

[22] The parameter γ is not separately identified if α and β are unconstrained. The estimates in Table 5 are obtained for $\gamma = 1$.

cospectrum pairs still implies strictly fewer degrees of freedom for the dependent variable than the raw data features. In other words, the higher statistical significance obtained for the spectral band regression is not an artefact of replicating data observations through the formation of currency pairs. The sample cospectrum terms are distinct sample observations with respect to the quadratic model so that standard maximum likelihood inference applies.

Table 5 reports the spectral band regressions for each of the three spectral bands. Panels A and B use the elasticity specification based on equity market weights, while Panels C and D proxy the exchange rate elasticity by the FX trading volume. For both specifications, different samples of exchange rate pairs are used. Panels A and C use the full sample of all $n = 595$ currency pairs, while Panels B and D estimate the regression for the $n = 231$ currency pairs consisting of the 22 most liquid currencies. A qualitatively similar result is obtained for all four panels. The regression for the high-frequency band in each panel features statistically highly significant coefficient estimates $\widehat{\alpha}$ for the premium effect and negative point estimate $\widehat{\beta}$ for the hedging effect. No significant relationship is found for the three other spectral bands.

For the high-frequency band the regression fit is very good, and particularly so for the subset of very liquid currency pairs in Panels B and D. In Panel B, the two independent variables explain 48 percent of the high-frequency cospectral shift of the 231 currency pairs. Figure 5 provides a graphical illustration of Table 5, Panel B, in which the high-frequency cospectrum shift $\Delta Cosp(i, j, H)$ is plotted against the (scaled) product $\gamma \times \Delta\widehat{e}_i \times \Delta\widehat{e}_j$ of predicted exchange rate changes. The t-values for the corresponding coefficient estimates $\widehat{\alpha}$ and $\widehat{\beta}$ are 7.02 and -28.27, respectively. Two currencies with a premium and hedge term such that their predicted forward rate changes are for example -2% and $+3\%$, respectively, should feature a predicted (high-frequency) cospectral shift of $-6 \times \gamma$, where $\gamma > 0$ is a positive scaling term.[23] The scatter plot in Figure 5 confirms a strong correlation between

[23] While γ is not separately identified in equation (16), it can be infered indirectly by matching

model implied spectral shifts and the observed cospectral shift in the high frequency band. It provides a model validation at a much higher level of statistical significance than the conventional evidence in Tables 2 and 3.

It is also instructive to compare the point estimates $\widehat{\alpha}$ and $\widehat{\beta}$ in Table 5 with the respective coefficients obtained in the traditional cross-sectional analysis. While the absolute coefficient size $\widehat{\alpha}$ and $\widehat{\beta}$ is not informative given the different dependent variable and the scaling term γ, the coefficient ratio $\widehat{\beta}/\widehat{\alpha}$ should be similar across inference methods. The best model fit for a 7-day window is obtained in Table 3, Panel A, for forward rates and capitalization based elasticities with a ratio $\widehat{\beta}/\widehat{\alpha} = -169$ ($\approx -54.21/0.32$). The corresponding ratio in Table 5, Panel B, is slightly more negative at $\widehat{\beta}/\widehat{\alpha} = -213$ ($\approx -627.23/2.94$), but has the same order of magnitude. The cross-sectional and spectral methods therefore give economically similar results. Yet their respective statistical significance is different: A much smaller standard errors in Table 5 make the spectral method the much sharper inference method and increase our confidence in the results.

For the economic interpretation we highlight that the standard deviation of the arbitrage risk is approximately 65 times smaller than that of the premium term. This implies that the exchange rate effects from the hedging terms are on average larger than those generated by the premium term. Hedging effects can therefore largely obscure the predicted premium changes unless an event study controls for the hedging terms.

Finally, two general implications of this MSCI event study can be highlighted. First, if speculative FX trading is widespread in the currency market, cross-currency hedging terms provide a plausible explanation for the exchange rate disconnect puzzle. The fundamental change corresponds to the premium change in our model, while the quantitatively important hedging term is generally ignored in macroeconomic specifications. In the light of our evi-

the average spectral shift in band H to the average produt of observed change rate changes; thus $\sum_i \sum_{j \neq i} |\Delta Cosp(i,j,H)| = \gamma \sum_i \sum_{j \neq i} |\Delta e_i \times \Delta e_j|$.

dence, the exchange rate disconnect puzzle might be a generalized model miss-specification which ignores the microeconomics of limited risk arbitrage. Second, carry trade strategies might have a considerable exchange rate impact which is only detectable if the correlation structure of all exchange rates is taken into account. More evidence on other speculative episodes seems desirable to confirm this conclusion. The spectral analysis represented here constitutes a very useful statistical tool to identify the structure and price impact of multi-asset trading strategies and make progress in this direction.

7 Conclusion

Currency trading strategies typically involve many currencies simultaneously so that a portfolio approach is the most appropriate analytical framework. In such a multi-currency setting, risk averse currency speculators can generate over- or undershooting across correlated currencies, thus generating an apparent "disconnect" from the currency fundamental (or premium) change to be arbitraged. Intuitively, the optimal risk arbitrage positions depend positively on the expected arbitrage premium, but negatively on the marginal risk contribution of any arbitrage position to overall arbitrage risk. Similarly, the price effect of arbitrage trading can be broken down into a premium (or fundamental) component and a transitory risk-hedging component.

A unique natural experiment is used to test the portfolio approach and the role of arbitrage risk hedging for short-run exchange rate movements. The redefinition of the MSCI global equity index in 2001 and 2002 was an exogenous shock to global equity allocations and generated predictable exchange rate returns. Two different data sets and statistical approaches are used to trace the impact of speculative arbitrage.

First, a conventional event study approach is used in which daily returns over different event windows are regressed on the premium and risk-hedging components. The premium effect is marginally significant with the predicted positive sign, while the evidence for the risk-hedging effect is much stronger. The evidence is confirmed for the subsample of forward rates that feature virtually identical event returns for the most common maturities. The point estimates indicate an economically significant currency effect for the transitory hedging demand of a 3.6 percent return difference for a two standard deviation change in the hedging benefit of a currency. But a clear limitation of the evidence is that the sample size of only 37 currencies combined with an event window of many days implies large standard errors for the point estimates.

The second part of the paper develops a new approach using high-frequency data combined with spectral analysis to obtain stronger statistical results. Additional statistical power comes from the identifying assumption that risk averse arbitrageurs undertake highly synchronized trades across currencies. Such synchronous implementation of a multi-currency arbitrage position implies a distinct shift in the high-frequency cospectrum across currency pairs. The cospectral shift should be proportional to the product of the predicted exchange rate changes. The spectral band regressions allow for a much more precise inference since "low-frequency noise" is filtered. The spectral approach provides very strong statistical evidence in favor of the model of limited arbitrage and in particular on the important role of hedging demands for the short-run exchange rate dynamics.

Overall, the multi-currency portfolio approach appears to correctly capture risk arbitrage behavior in the currency market. Speculators' risk aversion explains why the risk-hedging component is a very significant pricing factor over the arbitrage period. On a methodological level, I highlight that the increasing availability of high-frequency data allows for better statistical inference about multi-asset arbitrage strategies. The segmentation of the frequency domain can substitute for cross-sectional sample size and allow for sharper inference, as exemplified in this paper.

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Appendix

Proposition 1

Let e_t denote a vector of exchange rates $i = 1, 2, 3, ..., n$. The time interval $[0, T]$ is partitioned into N equal intervals $\Delta t = T/N$ and trading occurs at times $t = 0, \Delta t, 2\Delta t, 3\Delta t, ..., T$. Speculators learn about the supply shock $u = w^n - w^o$ at time $t = s < T$. The market clearing condition requires

$$\begin{array}{rclll} \mathbf{q}(e_t - \Phi_t + rt) & = & 0 & \text{for} & t < s \\ \mathbf{q}(e_t - \Phi_t + rt) & = & (\rho\Sigma\Delta t)^{-1}\mathcal{E}_1(e_{t+\Delta t} - e_t + r\Delta t) & \text{for} & s \leq t < T \\ \mathbf{q}(e_t - \Phi_t + rt & = & u & \text{for} & t = T, \end{array} \tag{17}$$

where $\mathbf{q}$ represents the diagonal matrix $(n \times n)$ with the currency-specific supply elasticities as elements. Taking differences between the equilibrium conditions for $t = T$ and $t = T - \Delta t$ and applying expectation operator $\mathcal{E}_{T-\Delta t}$ on both sides gives

$$\mathbf{q}\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} + r\Delta t) = u - (\rho\Sigma\Delta t)^{-1}\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} + r\Delta t) \tag{18}$$

and solving for the expected exchange rate return yields

$$\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} + r\Delta t) = \left[\mathbf{I} + (\mathbf{q}\rho\Sigma\Delta t)^{-1}\right]^{-1}\mathbf{q}^{-1}u. \tag{19}$$

Substitution of (19) into the market clearing condition (17) at time $t = T - \Delta t$ implies

$$\begin{aligned} e_{T-\Delta t} &= \Phi_{T-\Delta t} - r(T - \Delta t) + (\mathbf{q}\rho\Sigma\Delta t)^{-1}\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} - r\Delta t) \\ &= \Phi_{T-\Delta t} - r(T - \Delta t) + [\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1}\mathbf{q}^{-1}u. \end{aligned} \tag{20}$$

For a small Δt we can use the linear approximation $[\mathbf{I} - \mathbf{q}\rho\Sigma\Delta t] \approx [\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1}$and simplify

$$e_{T-\Delta t} \approx \Phi_{T-\Delta t} - r(T - \Delta t) + \mathbf{q}^{-1}u - \rho\Delta t(\Sigma u). \tag{21}$$

The equilibrium condition for the periods t with $s \leq t < T - \Delta t$ follows by repeated substitution. Starting with the market clearing condition (17) for $t = T - 2\Delta t$,

$$\mathbf{q}(e_{T-2\Delta t} - \Phi_{T-2\Delta t} + r(T - 2\Delta t)) = (\rho\Sigma\Delta t)^{-1}\mathcal{E}_{T-2\Delta t}(e_{T-\Delta t} - e_{T-2\Delta t} + r\Delta t), \tag{22}$$

substitution for $e_{T-\Delta t}$ yields

$$e_{T-2\Delta t} - \Phi_{T-2\Delta t} + r(T - 2\Delta t) = [\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1} (\mathbf{q}^{-1}u - \rho\Delta t(\Sigma u). \tag{23}$$

Using the approximation $[\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1} \approx [\mathbf{I} - \mathbf{q}\rho\Sigma\Delta t]$ again and ignoring terms of order $(\Delta t)^2$ implies

$$e_{T-2\Delta t} \approx \Phi_{T-2\Delta t} - r(T - 2\Delta t) + \mathbf{q}^{-1}u - \rho 2\Delta t(\Sigma u). \tag{24}$$

Repeated backward substitution for all t up to $t = s$ yields

$$e_s \approx \Phi_s - rs + \mathbf{q}^{-1}u - \rho(T - s)(\Sigma u). \tag{25}$$

The exchange rate change at time $t = s$ follows as

$$\Delta e_s = e_s - e_{s-\Delta t} \approx \mathbf{q}^{-1}u - \rho(T - s)(\Sigma u). \tag{26}$$

An exact solution can be determined in the limit case with $\Delta t \to 0$. This amounts to solving the system of first-order stochastic differential equations characterized by

$$de_t = -rdt + \rho\mathbf{q}\Sigma(e_t - \Phi_t - rt)dt + d\varepsilon_t, \tag{27}$$

with $\Phi_t = \int_{s=0}^{t} d\varepsilon_t$. Instead of a term $\rho(\Sigma u)(T - t)$ linear in t, the dynamic adjustment toward T is governed by a linear combination $\sum_{i=1}^{n} A_i e^{\lambda_i t}$, where the coefficients λ_i denote the eigenvalues of the matrix $\rho\mathbf{q}\Sigma$ and the boundary condition $e_T = \Phi_T - rT + \mathbf{q}^{-1}u$ holds. The solution in equation (25) represents a linear approximation to the exact limit case with $\Delta t \to 0$. At $t = T$ the two solutions coincide in levels and in the first time derivative. This means that the linear approximation is good as long as the risk arbitrage period $T - s$ is short and the eigenvalues λ_i are small. The eigenvalues are small if the risk aversion ρ is small.

Proposition 2:

So far it has been assumed that all arbitrageurs learn about the supply shock u simultaneously and acquire an arbitrage position instantaneously at time $t = s$. Consider now the case in which arbitrageurs built their arbitrage positions sequentially over trading rounds $s = 1, 2, ...S$ in the event window, while their cross-sectional trading is still synchronous. Assume $r = 0$ for the sake of simplicity. Let Δe_s^{Event} denote the exchange rate return process over the event period of S intervals and Δe_s^{Contol} the exchange rate return process for an equally long control period. In accordance with the model it is assumed that the exchange rate effect of a persistent speculative demand shock is linear in size and also persistent. In the absence of arbitrage trading, the n currency prices (in logs) follow random walks with returns $\Delta e_s = e_s - e_{s-1} = \varepsilon_s$ such that $\mathcal{E}(\varepsilon_s) = 0$ and $\mathcal{E}_{s-1}(\varepsilon_s \varepsilon_s') = \Sigma$. The return covariance between a currency pair (i, j) is denoted by Σ_{ij}. Under the null hypothesis of no speculative activity in the event period, the covariance change between the event and the control period is zero. Formally,

$$cov(\Delta e_{is}^{Event}, \Delta e_{js}^{Event}) - cov(\Delta e_{is}^{Control}, \Delta e_{js}^{Control}) = \Sigma_{ij} - \Sigma_{ij} = 0. \tag{28}$$

Similarly, if the stochastic process Δe_s is the same over the event and control period, then the difference of the respective cospectra $\Delta Cosp(i, j, f)$ should be zero for any currency pair and all frequencies f, i.e.

$$\Delta Cosp(i, j, f) = Cosp(i, j, f)^{Event} - Cosp(i, j, f)^{Control} = 0. \tag{29}$$

Consider next the case of speculative activity in the event period. A sequence $k = 1, 2, .., K < S$ of speculators trade (each once) sequentially in trading rounds $s(1), s(2), ..., s(K) \leq S$.

Each speculator has a relative size θ_k so that his price impact in trading round s is given by

$$\theta_k \Delta\widehat{e}_{s(k)} = \theta_k \left\{ \alpha \times \frac{1}{q_i}(w^n - w^o)_i + \beta \times [\Sigma(w^n - w^o)]_i \right\}. \tag{30}$$

The combined size of all traders is scaled to 1, hence $\sum_{k=1}^{K} \theta_k = 1$. Assuming that each speculator trades in separate trading round, the covariance for the event period follows as

$$cov(\Delta e_{is}^{Event}, \Delta e_{js}^{Event}) = \Sigma_{ij} + \frac{1}{S} \sum_{k=1}^{K} \theta_k^2 \left[\Delta\widehat{e}_{is(k)} \times \Delta\widehat{e}_{js(k)} \right]. \tag{31}$$

The covariance change between the event and control period therefore follows as

$$cov(\Delta e_{is}^{Event}, \Delta e_{js}^{Event}) - cov(\Delta e_{is}^{Control}, \Delta e_{js}^{Control}) = \frac{K}{S} \overline{\theta^2} \left[\Delta\widehat{e}_i \times \Delta\widehat{e}_j \right] = \gamma_{ij}(0), \tag{32}$$

where an expected (average) impact parameter is defined as $\overline{\theta^2} = \frac{1}{K} \sum_{k=1}^{K} \theta_k^2$.

Let $\gamma_{ij}(h)$ denote the covariance change corresponding to a lag of h trading rounds. The change in the sample analog of the cospectrum at a particular frequency numbered by f can be expressed as[24]

$$\Delta Cosp(i, j, f) = \frac{1}{2\pi} \sum_{h=-S+1}^{S-1} \gamma_{ij}(h) \cos(h\omega_f), \qquad \text{with } \omega_f = 2\pi f/S. \tag{33}$$

Assume that each arbitrageur k trades only once and his trading period $s(k)$ represents an independent draw from a uniform distribution over all S trading opportunities. The likelihood of two arbitrageurs trading at an interval of h periods ($0 < |h| \leq S - 1$) is given by $\frac{2}{S^2}(S - |h|)$. Moreover, for K arbitrageurs there are $K(K-1)/2$ pairs of arbitrageurs who could trade at interval h. The expected number of trading events for the two series Δe_{is} and Δe_{js} at lag $h \neq 0$ among K speculators follows as

$$\vartheta(h) = \frac{K(K-1)(S - |h|)}{S^2}. \tag{34}$$

[24]For details see Hamiltion (1994), pages 268-275.

A parameter defined as

$$\underline{\theta}^2 = \frac{1}{K(K-1)} \sum_{k=1}^{K} \sum_{l=1, l\neq k}^{K} \theta_k \theta_l, \tag{35}$$

characterizes the joint expected covariance impact of two different arbitrageurs ($l \neq k$). The event period covariance between Δe_{is} and Δe_{is-h} lagged by $h \neq 0$ trading rounds follows as

$$\gamma_{ij}(h) = cov(\Delta e_{is}^{Event}, \Delta e_{js-h}^{Event}) = \frac{\vartheta(h)}{S} \underline{\theta}^2 \left[\Delta \widehat{e}_i \times \Delta \widehat{e}_j\right] \tag{36}$$

The cospectrum (for $\omega_f = 2\pi f/S$) is characterized as

$$\begin{aligned}
\Delta Cosp(i,j,f) &= \frac{1}{2\pi} \sum_{h=-S+1}^{S-1} \gamma_{ij}(h) \cos(h\omega_f) = \frac{1}{2\pi} \gamma_{ij}(0) + \frac{1}{\pi} \sum_{h=1}^{S-1} \gamma_{ij}(h) \cos(h\omega_f) \\
&= \left\{ \frac{1}{2\pi} \frac{K}{S} \overline{\theta^2} + \frac{1}{\pi} \sum_{h=1}^{S-1} \frac{\underline{\theta}^2 K(K-1)(S-h)}{S^2} \cos(h\omega_f) \right\} \left[\Delta \widehat{e}_i \times \Delta \widehat{e}_j\right] \\
&\approx \frac{1}{2\pi} \frac{K}{S} \overline{\theta^2} \left[\Delta \widehat{e}_i \times \Delta \widehat{e}_j\right]
\end{aligned} \tag{37}$$

based on the approximation

$$\sum_{h=1}^{S-1} \frac{(S-h)}{S} \cos(h\omega_f) \approx \int_0^1 (1-x) \cos(2\pi f x) dx = 0. \tag{38}$$

Only the synchronized trading ($h = 0$) of the same speculator across two currencies makes any systematic contribution to the cospectrum change $\Delta Cosp(i,j,f)$, which is proportional to $\Delta \widehat{e}_i \times \Delta \widehat{e}_j$ for every frequency f. This implies that the high-frequency cospectrum shift $\Delta Cosp(i,j,\text{High})$ is also proportional to $\Delta \widehat{e}_i \times \Delta \widehat{e}_j$. The result is obtained under the assumption that speculative demand generates K persistent linear exchange rate effects without serial return correlation. Empirically, however, currency trading generates a negative serial correlation for currency returns. The midprice between the best ask and bid quotes tends to overshoot briefly. Under negative serial correlation, high-frequency components of the cospectrum capture a relatively larger proportion of the overall covariance in any currency pair. Hence, the speculative trading pattern is most pronounced in the highest spectral band.

Table 1: Summary Statistics on Exchange Rate Panel Data

Reported are summary statistics for the dependent and independent variables in Panels A and B, respectively. Panel data on daily exchange rate returns ($n = 37$) and forward rate returns for the 1-month forward rate ($n = 21$) are reported for the 7-day event window complemented by 2 years of return data from July 1, 1998 to July 1, 2000. The cospectrum shift is calculated for the 7-day event window relative to a 7-day control window using minute-by-minute return data from Olsen Associates for $n = 35$ currencies.

			Obs.	Mean	S.D.	Min	Max
Panel A: Dependent Variables							
Exchange rate returns		Δe_{it}	$15,096$	-0.035	0.584	-11.770	5.993
Forward rate returns		Δf_{it}	$8,477$	-0.029	0.612	-5.156	5.137
Cospectrum shift							
High-frequency band	$(\times 10^9)$	$\Delta Cosp(i, j,\text{High})$	595	0.865	17.60	-92.80	59.83
Medium-frequency band	$(\times 10^9)$	$\Delta Cosp(i, j,\text{Medium})$	595	-0.457	10.30	-58.18	51.02
Low-frequency band	$(\times 10^9)$	$\Delta Cosp(i, j,\text{Low})$	595	-1.823	8.35	-74.14	27.94
Panel B: Independent Variables							
Weight change		$(w^n - w^o)_i$	37	-0.0015	0.0055	-0.0289	0.0107
Absolute weight change		$\lvert w^n - w^o\rvert_i$	37	0.0021	0.0053	0.0000	0.0289
Elast. 1 × weight change		$\frac{2}{(w^n+w^o)_i} \times (w^n - w^o)_i$	37	-0.4314	0.3260	-0.9825	0.1086
Elast. 2 × weight change	$(\times 10^6)$	$\frac{1}{Vol_i^{FX}} \times (w^n - w^o)_i$	37	-0.3028	0.4009	-1.6812	0.0688
Marginal risk contribution		$[\Sigma(w^n - w^o)]_i$	37	-0.0041	0.0050	-0.0143	0.0010

Table 2: Panel Regressions for Daily Spot Exchange Rate Returns

The (log) daily spot exchange rate returns Δe_{it} (denominated in dollars per local currency and expressed in percentage points) is regressed on a constant, a time dummy D_t marking the event window, the time dummy interacted with the product of the supply elasticity q^{-1} and MSCI weight change $(w^n - w^o)_i$ of all stocks in currency i and the time dummy interacted with the risk contribution $[\Sigma(w^n - w^o)]_i$ of a currency to the arbitrage portfolio. Formally,

$$\Delta e_{it} = \lambda_0 + \lambda_1 \times D_t + \alpha \times D_t \times q_i^{-1}(w^n - w^o)_i + \beta \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it}, \quad E(\mu_t \mu_t') = \Omega.$$

The time period covers 2 years of daily exchange rate returns in 37 currencies for the period of July 1, 1998 to July 1, 2000 and the additional event window. Reported are results for event windows of 3, 5, and 7 trading days. Panel A reports results where the currency-specific elasticity q_i is proxied by the relative market capitalization, i.e. the average old and new index weights $q_i = \frac{1}{2}(w^n + w^o)_i$. Panel B proxies the same regressions where the elasticity is proxied by the currency specific trading volume according to the BIS 2001 triennial market survey, i.e. $q_i = Vol_i^{FX}$. The constant coefficient estimate a is not reported. Panel corrected z-values are reported in parenthesis. The adjusted R^2 states the explanatory power for the event window period. Statistical significance at the 5%, 3% and 1% level is marked by *, ** and ***, respectively.

Event Window	λ_1	[z]	α	[z]	β	[z]	Adj. R^2
Panel A: Capitalization-Based Exchange Rate Elasticities (Spot Rates Returns, N=37)							
3 Days	0.63^{***}	[4.43]	0.57^{***}	[3.54]			0.330
3 Days	0.19	[0.12]	0.24	[1.71]	-81.44^{***}	[−2.91]	0.545
5 Days	0.44^{***}	[3.93]	0.41^{***}	[3.30]			0.206
5 Days	0.05	[0.53]	1.12	[1.12]	-71.85^{***}	[−3.32]	0.433
7 Days	0.42^{***}	[4.46]	0.43^{***}	[4.04]			0.193
7 Days	0.10	[1.37]	0.20^{*}	[2.08]	-58.78^{***}	[−3.21]	0.354
Panel B: Volume-Based Exchange Rate Elasticities (Spot Rate Returns, N=37)							
3 Days	0.49^{***}	[3.87]	0.32^{***}	[3.20]			0.299
3 Days	0.12	[1.04]	0.13	[1.39]	-84.15^{***}	[−3.02]	0.539
5 Days	0.33^{***}	[3.32]	0.21^{***}	[2.68]			0.181
5 Days	0.00	[0.00]	0.04	[0.55]	-73.71^{***}	[−3.43]	0.430
7 Days	0.30^{***}	[3.64]	0.21^{***}	[3.16]			0.161
7 Days	0.03	[0.42]	0.07	[1.13]	-61.63^{***}	[−3.38]	0.346

Table 3: Panel Regressions for Daily Forward Rate Returns

The (log) daily returns of the 1-week forward FX rate Δf_{it} (denominated in dollars per local currency and expressed in percentage points) is regressed on a constant, a time dummy D_t marking the event window, the time dummy interacted with the MSCI weight change $(w^n - w^o)_i$ of all stocks in currency i and the time dummy interacted with the risk contribution $[\Sigma(w^n - w^o)]_i$ of a currency to the arbitrage portfolio. Formally,

$$\Delta f_{it} = \lambda_0 + \lambda_1 \times D_t + \alpha \times D_t \times q_i^{-1}(w^n - w^o)_i + \beta \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it}, \qquad E(\mu_t \mu_t') = \Omega.$$

The time period covers 2 years of daily forward rate returns in 22 currencies for the period of July 1, 1998 to July 1, 2000 and the additional event window. We report results for event windows of 3, 5, and 7 trading days. Panel A reports results where the currency specific elasticity q_i is proxied by the relative market capitalization, i.e. the average old and new index weights $q_i = \frac{1}{2}(w^n + w^o)_i$. Panel B reports the same regressions where the elasticity is proxied by the currency-specific trading volume according to the BIS 2001 triennial market survey, i.e. $q_i = Vol_i^{FX}$. The constant coefficient estimate a is not reported. Panel corrected z-values are reported in parentheses. The adjusted R^2 states the explanatory power for the event window period. Statistical significance at the 5%, 3% and 1% level is marked by *, ** and ***, respectively.

Event Window	λ_1	[z]	α	[z]	β	[z]	Adj. R^2
Panel A: Capitalization Based Exchange Rate Elasticities (Forward Rates, N=22)							
3 Days	0.82^{***}	[4.63]	0.65^{***}	[3.03]			0.452
3 Days	0.36^{**}	[2.26]	0.46^{**}	[2.25]	-73.10^{**}	[−2.42]	0.592
5 Days	0.60^{***}	[4.34]	0.51^{***}	[3.07]			0.325
5 Days	0.16	[1.32]	0.33^{*}	[2.10]	-68.57^{***}	[−2.93]	0.500
7 Days	0.54^{***}	[4.49]	0.46^{***}	[3.29]			0.292
7 Days	0.20	[1.89]	0.32^{**}	[2.42]	-54.21^{***}	[−2.74]	0.414
Panel B: Volume Based Exchange Rate Elasticities (Forward Rates, N=22)							
3 Days	0.68^{***}	[4.29]	0.42^{***}	[3.43]			0.429
3 Days	0.24	[1.54]	0.26^{**}	[2.18]	-75.27^{**}	[−2.47]	0.578
5 Days	0.50^{***}	[4.05]	0.39^{***}	[3.96]			0.312
5 Days	0.09	[0.75]	0.23^{**}	[2.46]	-69.58^{***}	[−2.95]	0.493
7 Days	0.45^{***}	[4.32]	0.34^{***}	[4.38]			0.282
7 Days	0.13	[1.27]	0.23^{***}	[3.01]	-55.17^{***}	[−2.76]	0.408

Table 4: Cospectrum Within and Across Currency Groups

The mean and standard deviation of the (scaled) cospectrum between currency returns is reported for four different spectral frequency bands B. In Panel A, currency pairs are drawn within groups $W+H+$ or $W-H-$, for which arbitrageurs are expected to trade in the same direction for both currencies. In Panel B, currency pairs are combined across groups where one currency is drawn from group $W+H+$ and the other from group $W-H-$. For the latter currency pairs arbitrage positions in opposite directions are expected, namely long and short positions, respectively. In Panel C, the sign of the cospectrum difference $\Delta Cosp(i,j,B)$ is reported for the 68 within-group currency pairs and the 72 cross-group currency pairs. The Fisher test reports the p-values for the null hypothesis that there is no association between the sign of the cospectrum change and the type of currency pair, namely within or across groups. The event period covers the 7 trading days from November 24 to December 4, 2000 and the control period the 7 trading days from September 8 to September 18. The weekend days (Saturday and Sunday) are excluded because of reduced trading activity. The Wilcoxon signed-rank test tests the null hypothesis that currency pairs have the same cospectrum during the event and control period. The high-frequency band aggregates currency comovements that occur within 15-minute intervals, the medium frequency band corresponds to co-movements from 15 minutes to 4 hours, and the low-frequency band sums up the remaining low frequencies.

Panel A: Cospectrum for Currency Pairs within Groups $W+H+$ and $W-H-$, N=68

	Event Period		Control Period		Difference		Signed-Rank Test	
Frequency Band B	Mean	S.D.	Mean	S.D.	Mean	S.D.	Z-Value	P-Value
High	17.36	18.71	8.97	13.47	8.39	9.35	5.116	0.0000
Medium	13.97	15.31	14.13	20.66	−0.16	8.61	−0.007	0.9947
Low	5.29	8.29	8.29	15.27	−3.00	7.67	−1.358	0.1746

Panel B: Cospectrum for Currency Pairs across Groups $W+H+$ and $W-H-$, N=72

	Event Period		Control Period		Difference		Signed-Rank Test	
Frequency Band B	Mean	S.D.	Mean	S.D.	Mean	S.D.	Z-Value	P-Value
High	−0.75	2.01	1.16	3.12	−1.91	3.56	−3.642	0.0003
Medium	0.05	1.39	−0.12	0.84	0.17	1.36	2.413	0.0158
Low	0.18	0.70	0.00	0.93	0.18	1.20	0.713	0.4760

Panel C: Sign of Cospectrum Difference $\Delta Cosp(i,j,B)$ for Within and Cross Group Currency Pairs

	Within Group Pairs			Cross Group Pairs			Fisher Test	
Frequency Band B	>0	≤ 0	All	>0	≤ 0	All	1-Sided	2-Sided
High	45	19	68	27	45	72	0.000	0.000
Medium	34	30	68	50	22	72	0.038	0.055
Low	33	31	68	39	33	72	0.448	0.864

Table 5: Spectral Band Regressions

The change in the cospectrum $\Delta Cosp(i,j,B)$ between the event period and the control period is calculated for different currency pairs (i,j) and three different spectral bands $B =$ High, Medium, Low. The frequency bands aggregate changes in currency return co-movements for periods of less than 15 minutes (High), from 15 minutes to 4 hours (Medium), and the remaining low frequencies (Low). The change in the cospectrum $\Delta Cosp(i,j,B)$ is explained by the quadratic form $\Delta\widehat{e}_i(\alpha,\beta) \times \Delta\widehat{e}_j(\alpha,\beta)$ in two parameters (α,β). Formally, we have a non-linear regression

$$\Delta Cosp(i,j,B) = \Delta\widehat{e}_i(\alpha,\beta) \times \Delta\widehat{e}_j(\alpha,\beta) + \epsilon_{ijB} \qquad \text{for } B = \text{High, Medium, Low},$$

where we define return functions in currencies i and j as

$$\Delta\widehat{e}_i(\alpha,\beta) = \alpha \times q_i^{-1}(w^n - w^o)_i + \beta \times [\Sigma(w^n - w^o)]_i$$
$$\Delta\widehat{e}_j(\alpha,\beta) = \alpha \times q_j^{-1}(w^n - w^o)_j + \beta \times [\Sigma(w^n - w^o)]_j.$$

Panels A and B use as elasticity parameter the average MSCI stock market capitalization, $q_i = \frac{1}{2}(w_i^n + w_i^o)$, and panels C and D the daily currency trading volume $q_i = Vol_i^{FX}$ according to the BIS 2001 triennial market survey. Panels A and C report the results for all currency pairs ($N = 595$), and Panels B and D only for the most liquid currency pairs that have liquid forward markets ($N = 231$). Statistical significance at the 5%, 3% and 1% level is marked by *, ** and ***, respectively

Frequency Band (B)	α	[t]	β	[t]	F-Test	Adj. R^2
Panel A: Capitalization Based Exchange Rate Elasticities, All Currency Pairs, N=595						
High	1.72***	[5.58]	−523.00***	[−24.61]	76.16	0.203
Medium	0.48	[0.25]	−10.44	[0.04]	0.01	0.003
Low	0.00	[0.00]	0.00	[0.00]	0.00	0.000
Panel B: Capitalization Based Exchange Rate Elasticities, Currency Pairs with Forward Rates, N=231						
High	2.94***	[7.02]	−627.03***	[−28.27]	108.42	0.482
Medium	0.00	[0.00]	0.00	[0.00]	0.00	0.000
Low	0.84	[0.29]	−27.72	[−0.11]	0.01	0.000
Panel C: Volume Based Exchange Rate Elasticities, All Currency Pairs, N=595						
High	1.44***	[4.41]	−491.79***	[−23.46]	91.50	0.188
Medium	0.97	[0.93]	−21.60	[−0.19]	0.11	0.003
Low	0.35	[0.16]	−7.78	[−0.03]	0.00	0.000
Panel D: Volume Based Exchange Rate Elasticities, Currency Pairs with Forward Rates, N=231						
High	1.60***	[3.43]	−555.05***	[−24.22]	78.02	0.400
Medium	2.42	[1.95]	−79.44	[−0.84]	0.53	0.000
Low	1.33	[0.72]	−29.18	[−0.20]	0.07	0.000

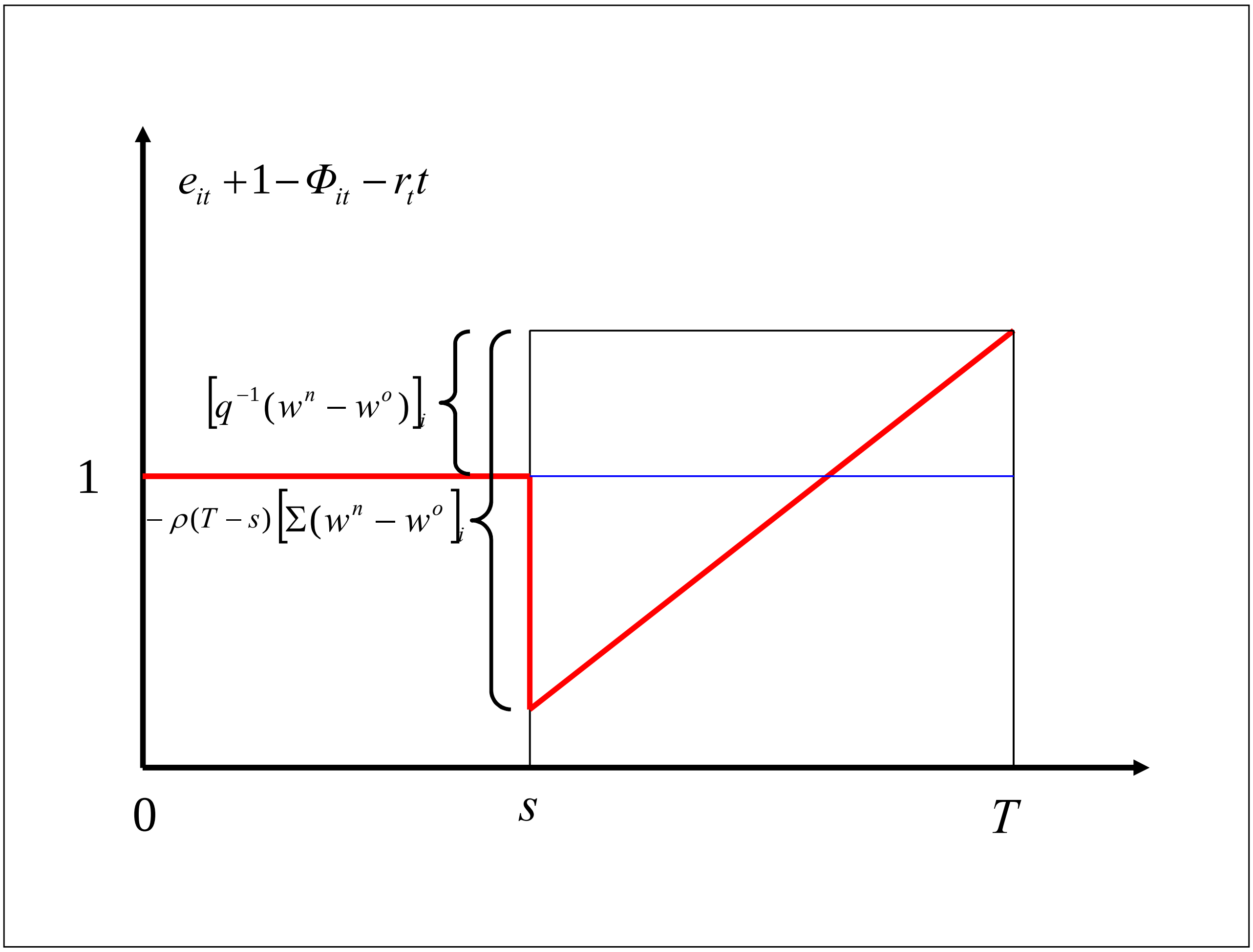

Figure 1: Plotted are the exchange rate dynamics under speculative arbitrage net of the fundamental process Φ_{it} and the effect of the interest differential rt. The fundamental effect at time $t = s$ is given by $\mathbf{q}^{-1}(w^n - w^o)$ and the risk hedging effect by $-\rho(T-s)\Sigma(w^n - w^o)$. Either over- or undershooting can be obtained depending on the sign of $[\Sigma(w^n - w^o)]_i$.

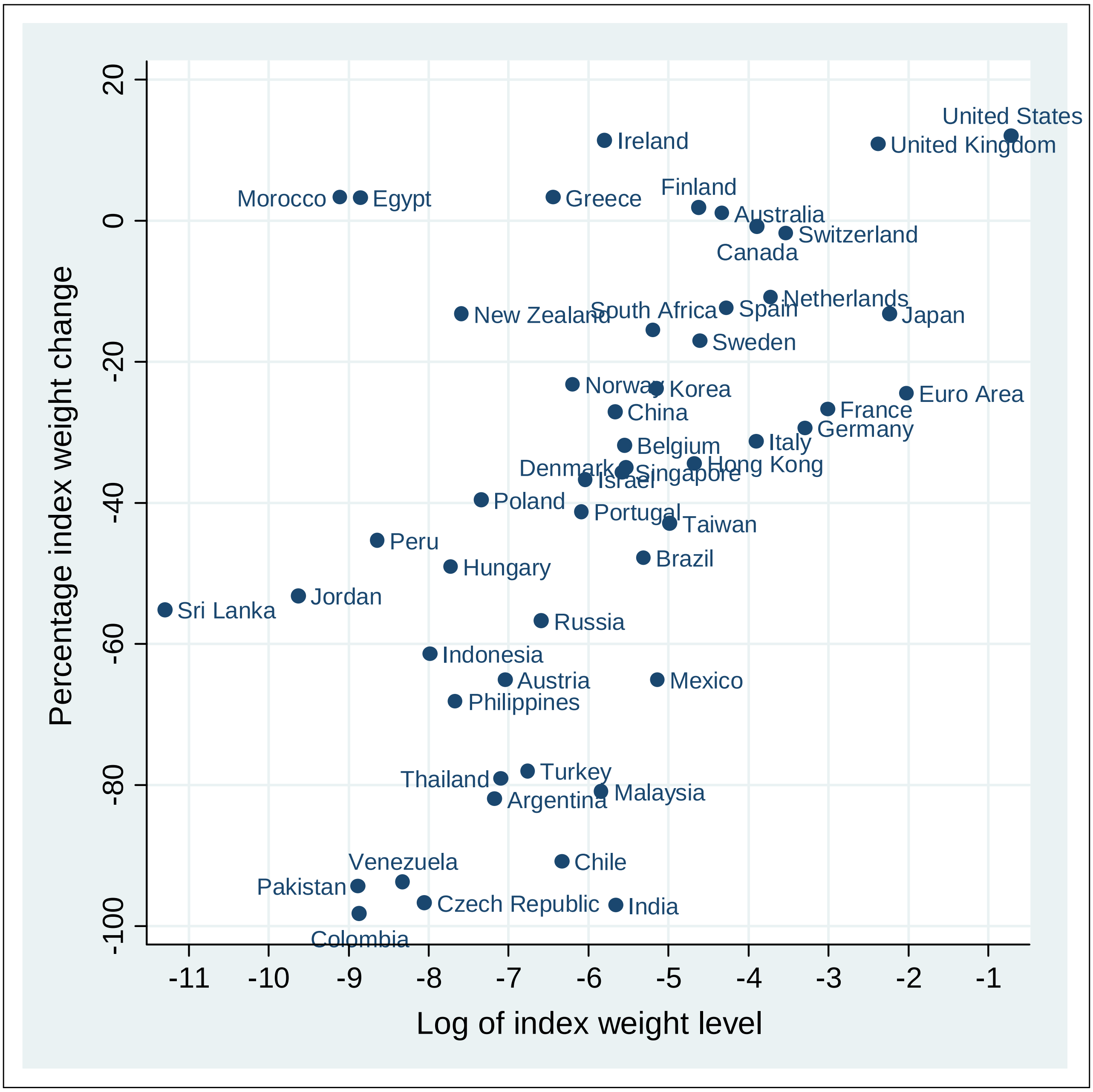

Figure 2: The percentage weight change $2(w^n - w^o)/(w^n + w^o)$ of each country in the MSCI global index is plotted against the log level of the old weight.

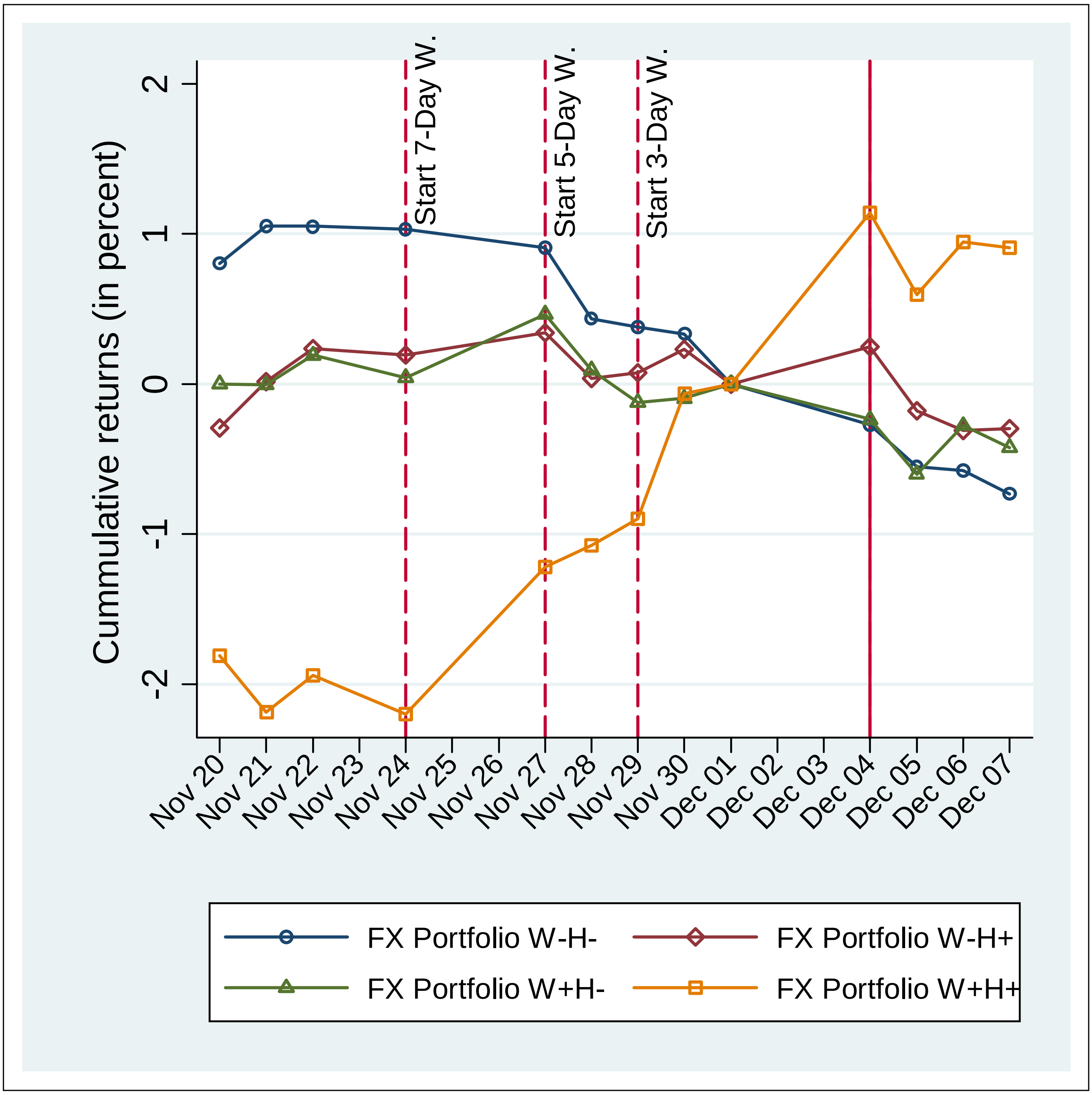

Figure 3: Currencies are sorted into those with above/below the median percentage weight change ($W+/W-$) and, in a second sort, into those with above/below median hedge value ($H+/H-$). All currencies in the sorted portfolio are equally weighted and their performance is plotted relative to an equally weighted currency portfolio composed of all 37 currencies. The dashed vertical lines mark the start of the 7-day, 5-day or 3-day event window, respectively, and the solid vertical line the end of all three event windows.

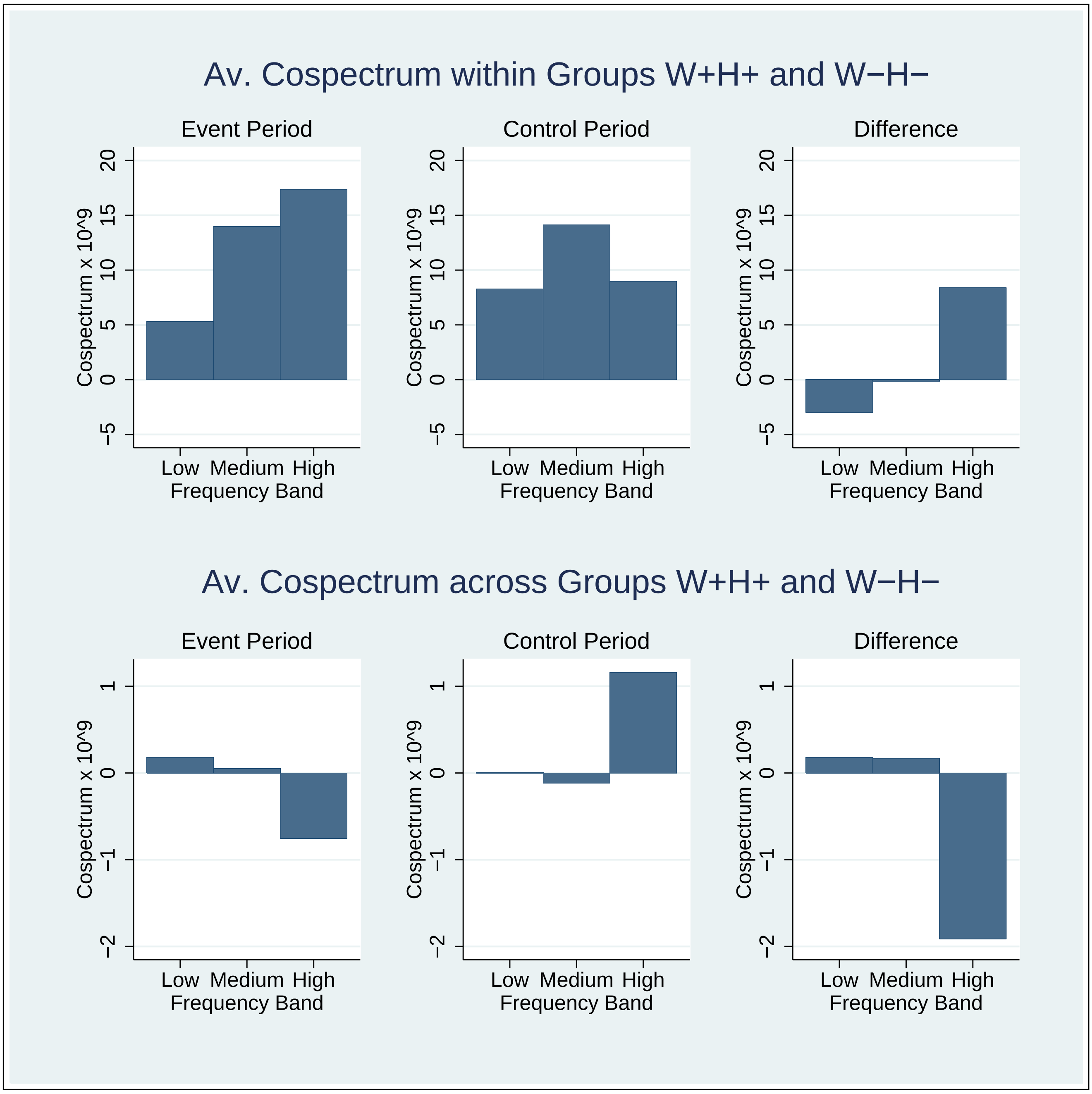

Figure 4: An average cospectrum is plotted for four different frequency bands B where the currency pairs are drawn (i) within the two groups $W+H+$ and $W-H-$ representing the most desirable and least desirable currencies and (ii) across groups where one currency is drawn from group $W+H+$ and the other from group $W-H-$. Column (1) graphs the average cospectrum $\overline{Cosp}(B)^{Event}$ over all pair permutations for the event period, column (2) the corresponding cospectrum $\overline{Cosp}(B)^{Control}$ for the control period, and column (3) documents the change $\Delta\overline{Cosp}(B)$ in the cospectrum. The high-frequency band aggregates currency co-movements that occur within 15-minute intervals, the medium-frequency band corresponds to co-movements from 15 minutes to 4 hours, and the low-frequency band sums up the remaining low frequencies.

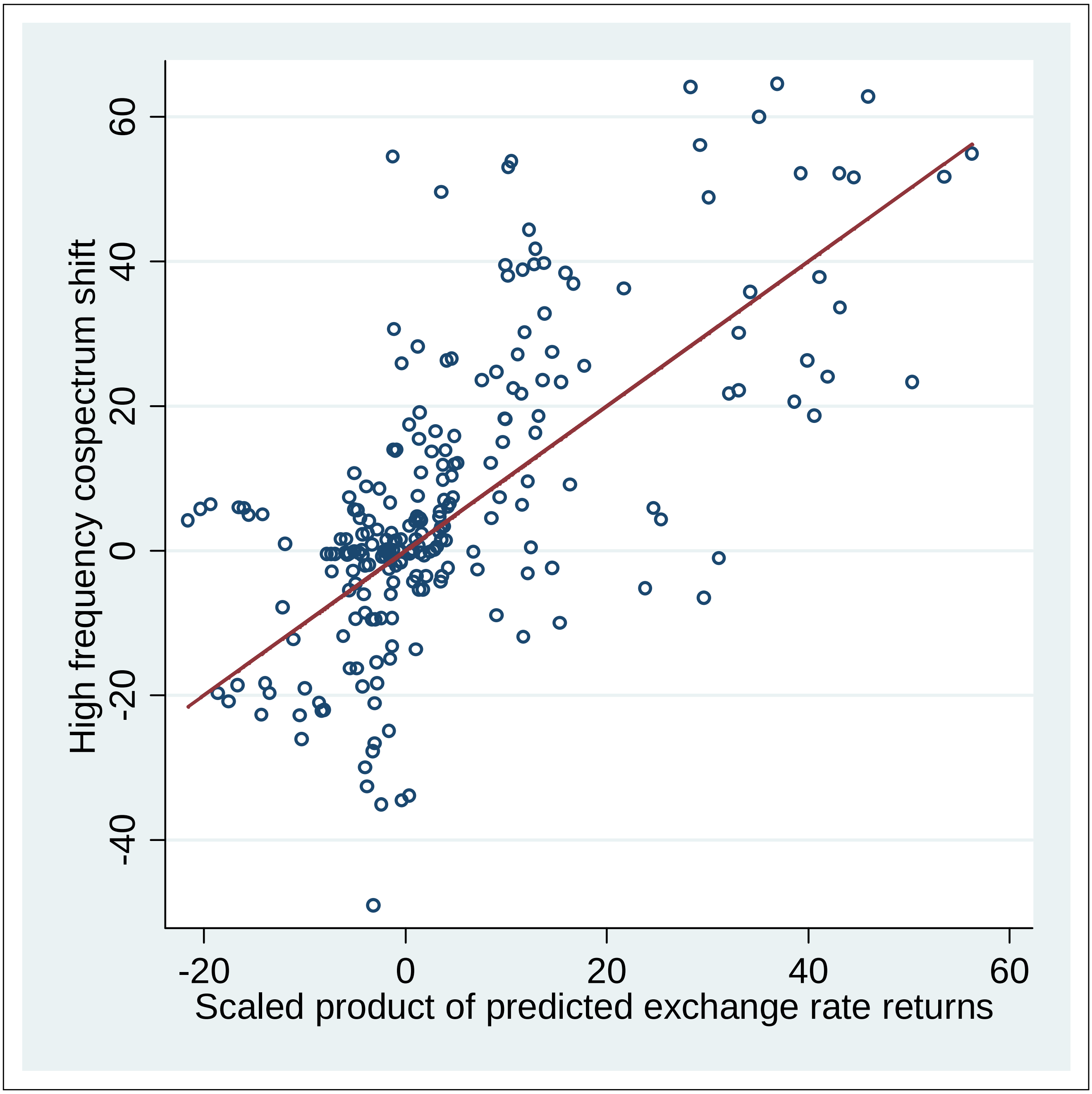

Figure 5: The shift of the high-frequency cospectrum $\Delta Cosp(i, j, \text{High})$ between all pairs formed by the 22 most liquid currencies is plotted against the (scaled) product $\gamma \times \Delta\widehat{e}_i \times \Delta\widehat{e}_j$ of the predicted event period returns induced by optimal arbitrage trading in currencies i and j.