Supplement
Can Portfolio Rebalancing Explain the Dynamics of Equity Returns, Equity Flows, and Exchange Rates?

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Abstract, Appendix, Figures

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Abstract

We explore whether international equity returns, equity portfolio flows, and exchange rate returns are consistent with the hypothesis that (unhedged) global investors rebalance their portfolio in order to limit their exchange rate exposure upon (1) relative equity return and (2) exchange rate shocks. We also explore whether (3) equity flow shocks influence the exchange rates and relative equity prices. In the estimation of the VAR system we do not impose any causal ordering upon the primitive shocks, but instead identify the system based on theoretical priors about the contemporaneous conditional correlations between the three variables. International data for the five largest equity markets are consistent with a theory in which equity returns and portfolio rebalancing are an important source of exchange rate dynamics.

Appendix

Assume the vector $y_t = (ER, FL, FX)'$ has an AR representation

$$A(L)\,(y_t - \phi) = \eta_t,$$

with $A(L) = 1 + A_1 L + A_2 L^2 + ... A_q L^q$, and 3×3 matrices $A_1, A_2, ... A_q$. Furthermore, $E(\eta_t \eta_t') = \Sigma$. Let the Wold MA representation be given by

$$y_t = \phi + A(L)^{-1}\eta_t = \phi + B(L)\eta_t$$

We first estimate the coefficients $\widehat{B}(L)$ and the matrix $\widehat{\Sigma}$. One possible decomposition of the matrix $\widehat{\Sigma} = PP'$ is the Choleski decomposition where P is a lower triangle matrix. For orthogonal innovations, $e_t \sim (0, I)$ with $e_t = P^{-1}\eta_t$, we have $E(\eta_t \eta_t') = E(P e_t e_t' P') = E(PP') = \Sigma$, and $C(L) = B(L)P$ is one possible MA representation of y_t.

$$y_t - \phi = B(L)\eta_t = B(L)PP^{-1}\eta_t = B(L)P\ e_t = C(L)e_t$$

Generally, we want to search over the set of all matrices $\widetilde{P} = PR$ which form a valid MA representation with $PR(PR)' = \widehat{\Sigma}$. This search is carried out through a combination of Jacobi rotations. We can define three distinct Jacobi rotations matrices (for $-\pi/2 < \theta_i < \pi/2$) as

$$\begin{aligned}
R_{\theta_1} &= \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
R_{\theta_2} &= \begin{pmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) \\ 0 & 1 & 0 \\ \sin(\theta_2) & 0 & \cos(\theta_2) \end{pmatrix} \\
R_{\theta_3} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_3) & -\sin(\theta_3) \\ 0 & \sin(\theta_3) & \cos(\theta_3) \end{pmatrix}
\end{aligned}$$

Any joint rotation $R = R_{\theta_1} R_{\theta_2} R_{\theta_3}$ fulfills $RR' = I$. Hence, PR also represents a decomposition of $\widehat{\Sigma}_0$ into primitive shock and $C(L) = B(L)PR$ the corresponding MA representation of y_t.

These different MA representations imply different impulse response functions and different correlation structures between the variables of y_t. Let s be a vector picking the impulse response to a particular primitive shock e_t (for example $s_1 = (1, 0, 0)$ for e_{1t}). The correlation of variable y_{it} and y_{jt} conditional on a shock of type s follows as

$$\rho_{ij|s}(R) = \frac{(C^i(L)s)(C^j(L)s)}{\sqrt{(C^i(L)s)^2 (C^j(L)s)^2}}$$

where $C^i(L)$ denotes the row i of $C(L)$. Given the definition of the vector y_t, correlations conditional on return shocks have $s_1 = (1, 0, 0)$, those conditional on flow shocks have $s_2 = (0, 1, 0)$, and those conditional on exchange rate shocks have $s_3 = (0, 0, 1)$.

Economic theory provides prior information about $k = 1, 2, ..6$ conditional correlations ρ_k. In particular it allows us to restrict the sign of ρ_k. We can therefore define a penalty function $f(.)$ which assigns a positive weight to MA representations in violation of theoretical sign restrictions and a negative weight if they are fulfilled. We concentrate here on a linear penalty function which gives a penalty of $f(k, R) = -\rho_{ij|s}(R)$ whenever a positive correlation is predicted and a penalty of $f(k, R) = \rho_{ij|s}(R)$ if a negative correlation is predicted. We then define m^3 grid points (with $m = 90$) for rotation angles $(\theta_1, \theta_2, \theta_3) \in [-\pi/2, \pi/2]^3$ and find the rotation R^* on the grid which minimizes the overall penalty. Formally,

$$R^* = \arg\min_{\{R\}} \sum_{k=1}^{6} f(k, R).$$

We then report the impulse response for the MA representation given by $C(L) = B(L)PR^*$.

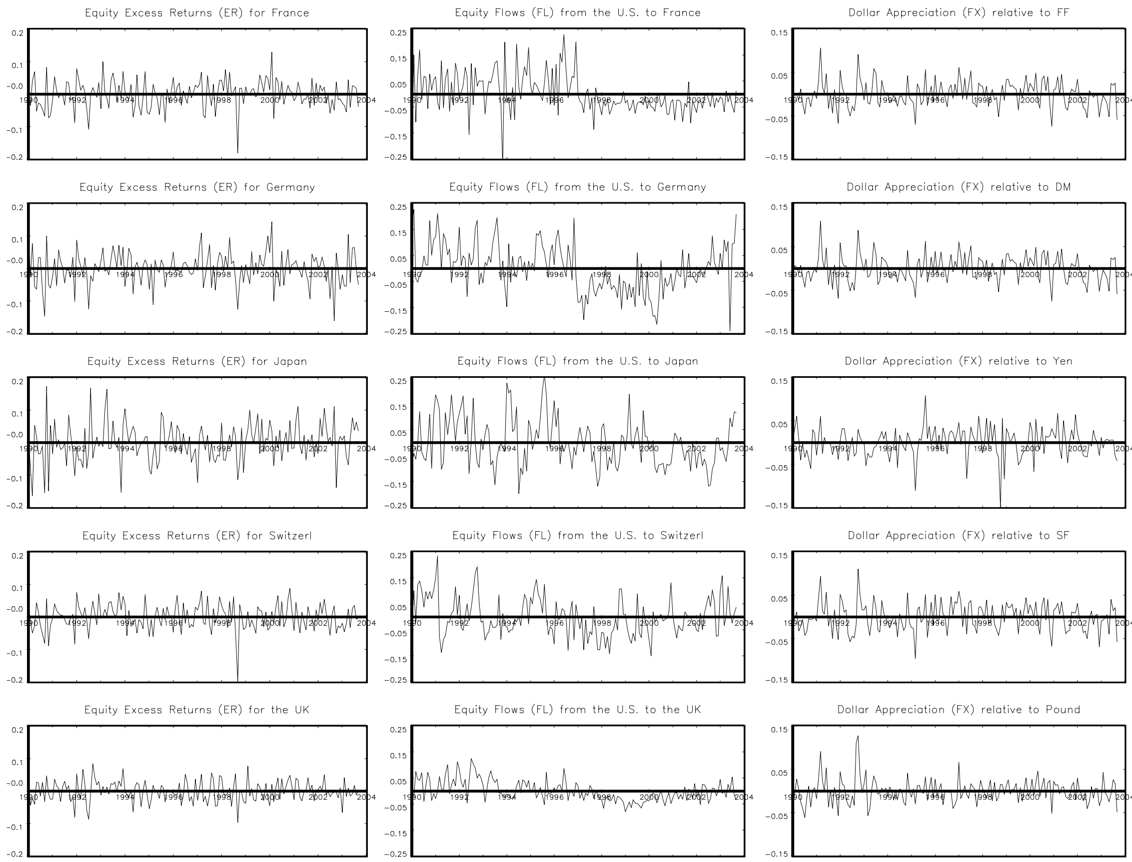

Figure 1: Shown are the monthly foreign equity excess returns (foreign market index return minus U.S. index return), the standardized U.S. equity outflows into the foreign country and the exchange rate returns (dollar appreciation is a positive return) for 5 countries, namely France, Germany, Japan, Switzerland and the U.K. The data period is January 1990 to September 2003.

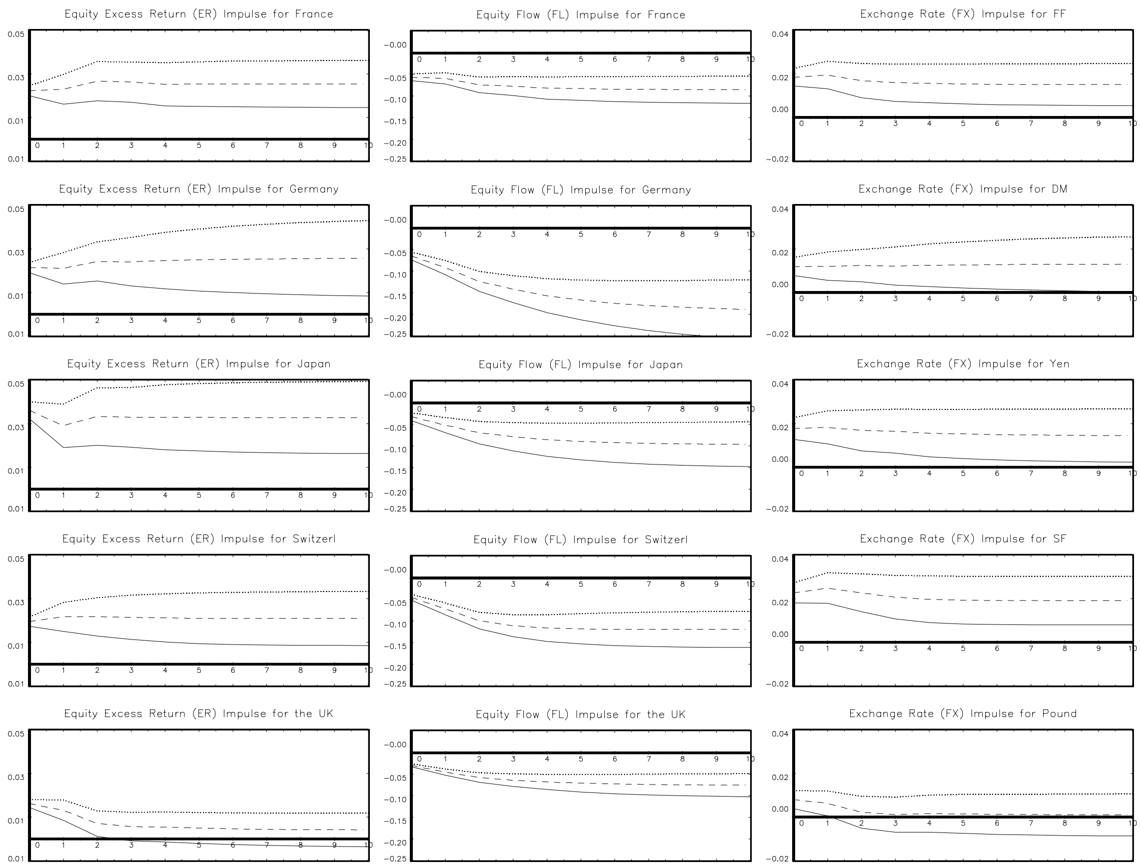

Figure 2: Shown are cumulative impulse response functions over 10 months of a foreign equity excess return shock on equity excess returns (column 1), U.S. equity outflows into the foreign country (column 2), and the exchange rate return (column 3) for the 5 sample countries (by row). Confidence intervals of 2 standard deviations are added.

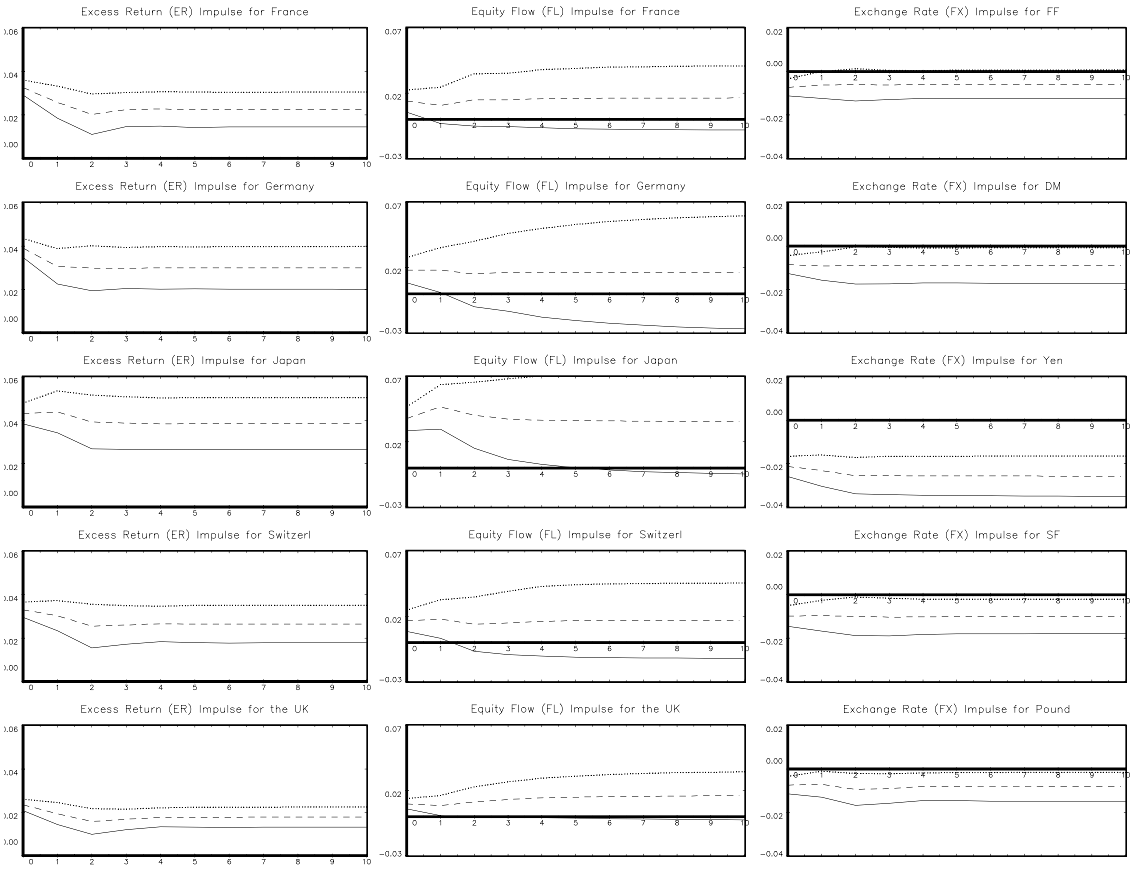

Figure 3: Shown are cumulative impulse response functions over 10 months of an equity outflow shock on foreign equity excess returns (column 1), U.S. equity outflows into the foreign country (column 2), and the exchange rate return (column 3) for the 5 sample countries (by row). Confidence intervals of 2 standard deviations are added.

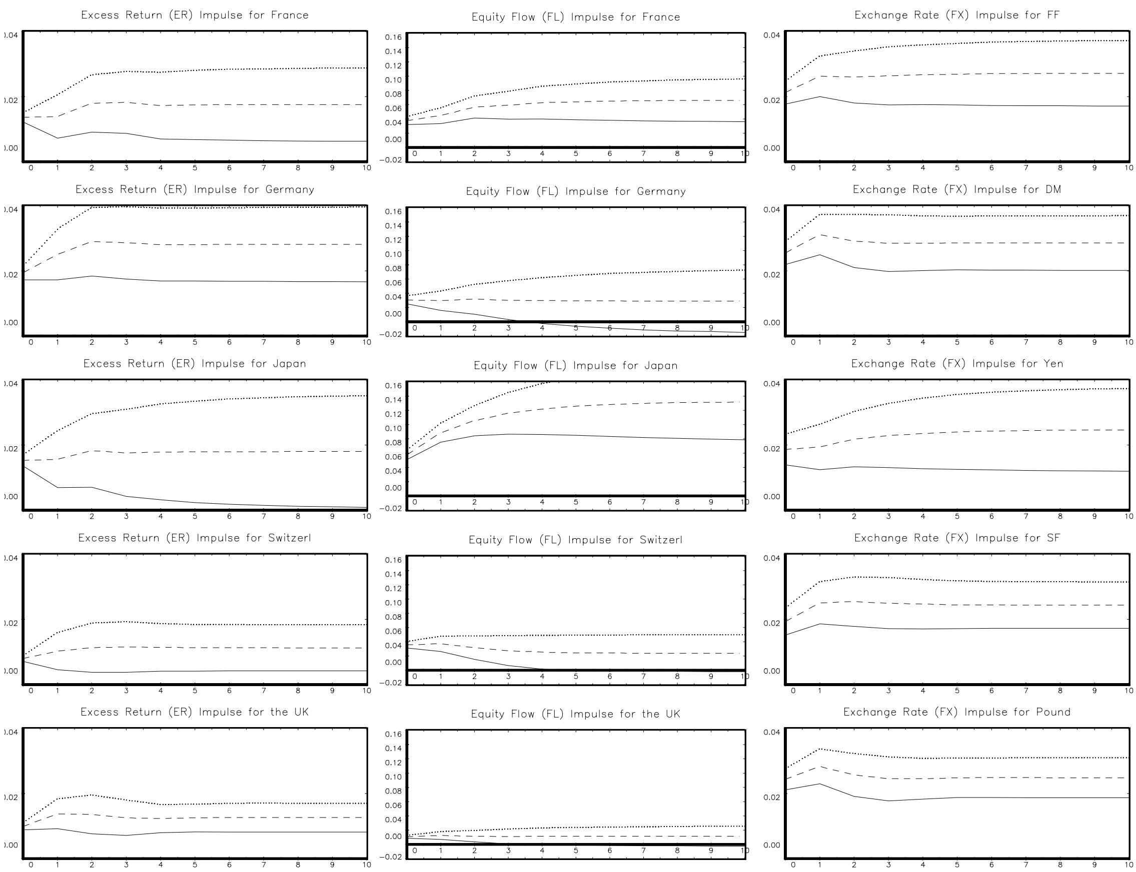

Figure 4: Shown are cumulative impulse response functions over 10 months of a FX return shock on foreign equity excess returns (column 1), U.S. equity outflows into the foreign country (column 2), and the exchange rate return itself (column 3) for the 5 sample countries (by row). Confidence intervals of 2 standard deviations are added.